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editorial

A *Revista Brasileira de Ciências Mecânicas - RBCM* completou, com o presente número, dez anos de existência. Durante este período a RBCM publicou 156 trabalhos em diversas áreas de engenharia, física e matemática aplicada. Um índice geral destes trabalhos é apresentado nas páginas subsequentes.

A criação e manutenção de uma Revista cujo objetivo seja publicar trabalhos de interesse permanente é uma tarefa árdua. Tal afirmação é particularmente verdadeira tanto no que concerne ao processo de editoração como à revisão e submissão de trabalhos por parte da comunidade científica que suporta a Revista. Neste contexto a RBCM representa os esforços de pesquisadores brasileiros que atuam em Ciências Mecânicas e que reconhecem a importância de haver um veículo de divulgação nacional nesta área. Graças a este esforço é que a RBCM existe, e se constitui hoje parte da memória técnico-científica brasileira.

Embora existam atualmente no mundo em torno de 35000 periódicos científicos cujo objetivo é disseminar e arquivar os avanços do conhecimento, a participação brasileira neste processo tem sido irrisória. O Brasil além de gerar pouco conhecimento científico devido à pequena parcela de sua sociedade que trabalha em pesquisa, não conseguiu que um grande número de seus pesquisadores se convencesse da importância de ter seus trabalhos registrados em artigos de interesse permanente. Poucos são os pesquisadores brasileiros que têm como hábito publicar sistematicamente em revistas o resultado de suas pesquisas. Este fato é facilmente confirmado ao se verificar que a RBCM é um dos únicos (se não o único) periódicos brasileiros na sua área, e que muitos pesquisadores atuantes em Ciências Mecânicas além de não fazerem uso da RBCM não têm seus nomes encontrados nas revistas estrangeiras correspondentes.

A publicação de um artigo em uma revista científica é uma atividade enriquecedora para os editores, revisores, para a comunidade que lê a revista, e, principalmente, para os autores. Um dos objetivos da RBCM para o futuro próximo é educar o pesquisador brasileiro neste sentido. A RBCM precisa ser utilizada e divulgada. Aqueles que se iniciam na atividade técnico-científica se tiverem sido educados a utilizar a RBCM, sem dúvida sentirão a necessidade de nela participar como autores. É um forte desejo dos presentes editores que o editorial dos vinte anos de existência da RBCM reflita este espírito.

Álvaro Toubes Prata
Editor Associado

UNSTEADY FORCED AND FREE CONVECTIVE FLOW PAST AN INFINITE VERTICAL POROUS PLATE WITH VARIABLE SUCTION AND OSCILLATORY WALL TEMPERATURE

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ABSTRACT

An analysis of the unsteady flow of an incompressible, viscous fluid past an infinite vertical plate has been carried out under the following conditions: 1) variable suction at the plate, 2) wall temperature oscillating about a non-zero constant mean, 3) constant free-stream. Approximate solutions to the coupled non-linear equations have been derived for the transient velocity and the transient temperature, the amplitude and phase of the skin-friction and the rate of heat transfer. The transient velocity and temperature are shown on graph whereas the numerical values of the amplitude of the skin-friction $|B|$ *and the amplitude of the rate of heat transfer* $|Q|$, *the phase of the skin-friction* $\tan\alpha$ *and the phase of the rate of heat transfer* $\tan\beta$ *are entered in the Table. The effects of* A *(variable suction parameter),* ω *(frequency),* G *(Grashof number) and* E *(Eckert number) are discussed.*

NOMENCLATURE

$\|B\|$	amplitude of skin/friction
c_p	specific heat at constant pressure
E	Eckert number
g	acceleration due to gravity
G	Grashof number

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k	thermal conductivity
M_r, M_i	fluctuating parts of the velocity profile
P	Prandtl number
q'	rate of heat transfer
$\|Q\|$	amplitude of the rate of heat transfer
t'	dimensional time
t	dimensionless time
T'	temperature of fluid
T'_w	temperature of the plate
T'_∞	temperature of the fluid in the free-stream
T_r, T_i	fluctuating parts of the temperature profile
u', v'	velocity components in x and y directions
u	dimensionless velocity in the x-direction
U_o	free-stream velocity
u_o	mean velocity
u_1	fluctuating part of the velocity
v_o	suction velocity
v', y'	co-ordinate system
y	dimensionless co-ordinate normal to the wall
ω'	frequency
ω	dimensionless frequency
τ	skin-friction
θ	dimensionless temperature
θ_o	mean temperature
$\varepsilon(\theta_1)$	amplitude of temperature fluctuation
α	phase angle of the skin-friction
β_1	phase angle of the rate of heat transfer
ζ'	density of the fluid
ζ'_∞	density of the fluid in the free-stream
ν	kinematic viscosity
μ	viscosity

INTRODUCTION

Oscillatory flows were studied by Moore [1], Lighthill [2], Lin [3]. In Ref. [2], small amplitude phenomenon was studied and in Ref. [3], finite-amplitude phenomenon was studied. The theoretical predictions were confirmed experimentally by Hill and Stenning [4]. Lighthill solved the problem by integral-method whereas Nickerson [5] solved it by numerical method. In the presence of constant suction at the horizontal plate, the effects of free-stream oscillations were studied by Stuart [6] and he observed that reverse-type of flow exists near the plate at high frequency. Messiha [7] studied Stuart's problem on taking into account variable suction at the horizontal plate. The effects of free-stream oscillations and the free-convection currents on the flow in the upward direction past an infinite vertical porous plate were studied by Soundalgekar [8,9] in case of constant suction and in the presence of variable suction by Soundalgekar [10]. In Refs. [7-10], the plate temperature was assumed to be isothermal.

Another physical situation which is often encountered in industrial applications is that the plate temperature is often found to be oscillating about a constant non-zero mean. Hence it is interesting to study these effects of oscillating plate temperature on the upward motion of a viscous fluid, past an infinite vertical porous plate, in the presence of an uniform free-stream. This was studied in the presence of constant suction by Soundalgekar [11] and it is now proposed to study the effects of variable suction at the vertical infinite porous plate in the presence of an uniform free-stream and oscillating plate temperature. In Section 2, the mathematical analysis is presented followed by discussion and in Section 3, the conclusions have been set out.

These results may be found applicable in air-conditioning, solar-energy applications, etc.

MATHEMATICAL ANALYSIS

Here the x'-axis is chosen along an infinite vertical porous plate in the vertically upward direction and the y'-axis is chosen normal to the plate. The unsteady flow of a viscous incompressible fluid past an infinite vertical porous plate in the upward direction is assumed. The mean plate temperature is assumed to be T'_w. Then the physical variables are functions of t' and y' only and are independent of x'. Then the problem is now governed by the following equations under usual Boussinesq's approximation.

Continuity:

$$\frac{\partial v'}{\partial y'} = 0 \qquad (1)$$

Momentum:

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \frac{\partial U'}{\partial t'} + g\beta(T'-T'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} \tag{2}$$

Energy:

$$\zeta' c_p \left(\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'}\right) = k \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'}\right)^2 \tag{3}$$

All the physical quantities have been defined in nomenclature.

The boundary conditions are:

$$\begin{aligned} &u' = 0 , \quad T' = T'_w + \varepsilon(T'_w - T'_\infty)e^{i\omega' t'} \qquad \text{at } y' = 0 \\ &u' = U' = U_o , \quad T' = T'_\infty \qquad \text{as } y' \to \infty \end{aligned} \tag{4}$$

For variable suction, equation (1) integrates to [Messiha [6]]

$$v' = -v_o\left[1+\varepsilon \ A \ e^{i\omega' t'}\right] \tag{5}$$

where A and C are small such that εA<1. A is called variable suction parameter. The negative sign in (5) indicates that the suction is directed towards the plate. Introducing the following non-dimensional quantities

$$\begin{aligned} &y = y' v_o/\nu , \quad t = t' v_o^2/4\nu , \quad w = 4\nu\omega'/v_o^2 \\ &u = u'/U_o , \quad U = U'/U_o , \quad \theta = \frac{T'-T'_\infty}{T'_w-T'_\infty} , \\ &P = \mu \ c_p/k , \quad G = \nu g\beta(T'_w-T'_\infty)/U_o v_o , \\ &E = U_o^2/c_p(T'_w-T'_\infty) \end{aligned} \tag{6}$$

in equations (2)-(4) and taking into account equation (5), we get

$$\frac{1}{4}\frac{\partial i}{\partial t} - \left(1+\varepsilon \ A \ e^{i\omega t}\right) \frac{\partial u}{\partial y} = G \ \theta + \frac{\partial^2 u}{\partial y^2} \tag{7}$$

$$\frac{P}{4}\frac{\partial \theta}{\partial t} - P\left(1+\varepsilon \ A \ e^{i\omega t}\right) \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PE \left(\frac{\partial u}{\partial y}\right)^2 \tag{8}$$

and the boundary conditions are

$$\begin{aligned} u = 0 \;, \quad \theta = 1 + \varepsilon\, e^{i\omega t} \qquad & \text{at} \;\; y = 0 \\ u = 1 \;, \quad \theta = 0 \qquad & \text{as} \;\; y \to 0 \end{aligned} \tag{9}$$

We observe from equations (7) and (8) that the problem is now governed by coupled non-linear equations. The exact solution to this system of equations is not possible. So we now derive an approximate solution. Hence following Lighthill [2], we assume for the velocity and temperature, in the neighbourhood of the plate, the following:

$$\begin{aligned} u &= u_o + \varepsilon\, e^{i\omega t}\, u_1 \\ \theta &= \theta_o + \varepsilon\, e^{i\omega t}\, \theta_1 \end{aligned} \tag{10}$$

where $\varepsilon < 1$. Substituting (10) in equations (7)-(8), equating the harmonic and the non-harmonic terms, neglecting the coefficients of ε^2, we get

$$u_o'' + u_o' = -G\theta_o \tag{11}$$

$$u_1'' + u_o' - \frac{i\omega}{4}\, u_1 = A u_o'^2 - G\theta_1 \tag{12}$$

$$\theta_o'' + P\theta_o' = -PE\, u_o'^2 \tag{13}$$

$$\theta_1'' + P\theta_1' - \frac{i\omega p}{4}\, \theta_1 = -PA\, \theta_o' - 2PE\, u_o'\, u_1' \tag{14}$$

and the boundary conditions are

$$\begin{aligned} u_o = 0 \,,\;\; u_1 = 0 \,,\;\; \theta_o = 1 \,,\;\; \theta_1 = 1 \qquad & \text{at} \;\; u = 0 \\ u_o = 1 \,,\;\; u_1 = 0 \,,\;\; \theta_o = 0 \,,\;\; \theta_1 = 0 \qquad & \text{as} \;\; y \to \infty \end{aligned} \tag{15}$$

Here and henceforth the primes denote differentiation with respect to y. The system of equations (11)-(14) is still coupled and non-linear and hence closed-form solutions are not possible. So to get approximate solutions, we expand u_o, u_1, θ_o, θ_1 in powers of E, the Eckert number, for E is always small in case of an incompressible fluids. Hence we assume

$$\begin{aligned} u_o = u_{o1} + E\, u_{o2} \;, \quad & u_1 = u_{11} + E\, u_{12} \\ \theta_o = \theta_{o1} + E\, \theta_{o2} \;, \quad & \theta_1 = \theta_{11} + E\, \theta_{12} \end{aligned} \tag{16}$$

Substituting (16) in equations (11)-(15), equating the coefficients of different powers of E, neglecting those of E^2, we get the following system of equations:

$$u''_{o1} + u'_{o1} = -G\theta_{o1} \tag{17}$$

$$u''_{o2} + u'_{o2} = -G\theta_{o2} \tag{18}$$

$$u''_{11} + u'_{11} - \frac{i\omega}{4} u_{11} = -A\, u'_{o1} - G\theta_{o1} \tag{19}$$

$$u''_{12} + u'_{12} - \frac{i\omega}{4} u_{12} = -A\, u'_{o2} - G\theta_{o2} \tag{20}$$

$$\theta''_{o1} + P\theta'_{o1} = 0 \tag{21}$$

$$\theta''_{o2} + P\theta'_{o2} = -P\, u'^{2}_{o1} \tag{22}$$

$$\theta''_{11} + P\theta'_{11} - \frac{i\omega p}{4} \theta_{11} = -PA\, \theta'_{o1} \tag{23}$$

$$\theta''_{12} + P\theta'_{12} - \frac{i\omega p}{4} \theta_{12} = -PA\, \theta'_{o2} - 2Pu'_{o1}u'_{11} \tag{24}$$

and the boundary conditions are

$$\left.\begin{array}{l} u_{o1} = 0\,,\ u_{o2} = 0\,,\ u_{11} = 0\,,\ u_{12} = 0 \\ \theta_{o1} = 1\,,\ \theta_{o2} = 0\,,\ \theta_{11} = 1\,,\ \theta_{12} = 0 \end{array}\right\} \quad \text{at } y = 0$$

$$\left.\begin{array}{l} u_{o1} = 1\,,\ u_{o2} = 0\,,\ u_{11} = 0\,,\ u_{12} = 0 \\ \theta_{o1} = 0\,,\ \theta_{o2} = 0\,,\ \theta_{11} = 0\,,\ \theta_{12} = 0 \end{array}\right\} \quad \text{at } y \to \infty \tag{25}$$

Equations (17)-(24) are coupled and linear equations. These are solved subject to boundary conditions (25). To save space, the solutions are not shown here. The mean velocity and the mean temperature are already shown in Soundalgekar [11]. Hence the expression for the transient velocity and the transient temperature can be written for $\omega t = \pi/2$ as

$$\begin{array}{l} u = u_o - \varepsilon M_i \\ \theta = \theta_o - \varepsilon\theta_i \end{array} \tag{26}$$

where

$$M_r + iM_i = u_1\,, \quad \theta_r + i\theta_i = \theta_1$$

The expressions are evaluated for different values of A, ω, G and E and the profiles are shown in Figures 1 and 2 for velocity and temperature respectively. We observe from this Figure 1 that due to the presence of variable suction, the transient velocity and temperature decrease and due to further increase in A, the velocity and temperature decrease more. An increase in the frequency leads to a decrease in the transient velocity, whereas an increase in G or due to greater viscous dissipative heat, there is an increase in the transient velocity and temperature.

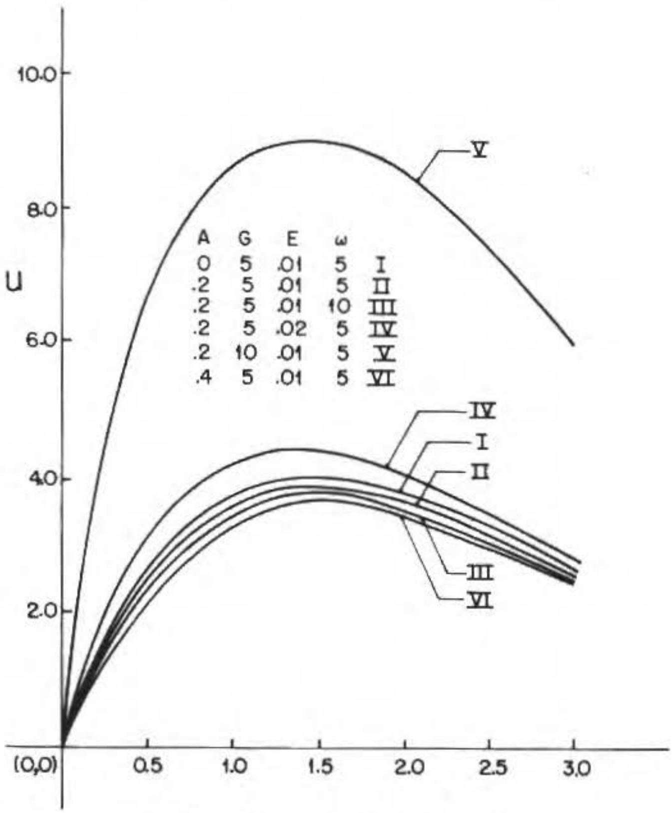

Figure 1. Transient velocity profiles
(ε=0.2, $\omega t=\pi/2$, p=0.72)

Knowing the velocity field, we now calculate the amplitude and the phase of the skin-friction. It is given by (Ref. [11])

$$\tau = \tau_m + C|B| \cos(\omega t+\alpha) \tag{27}$$

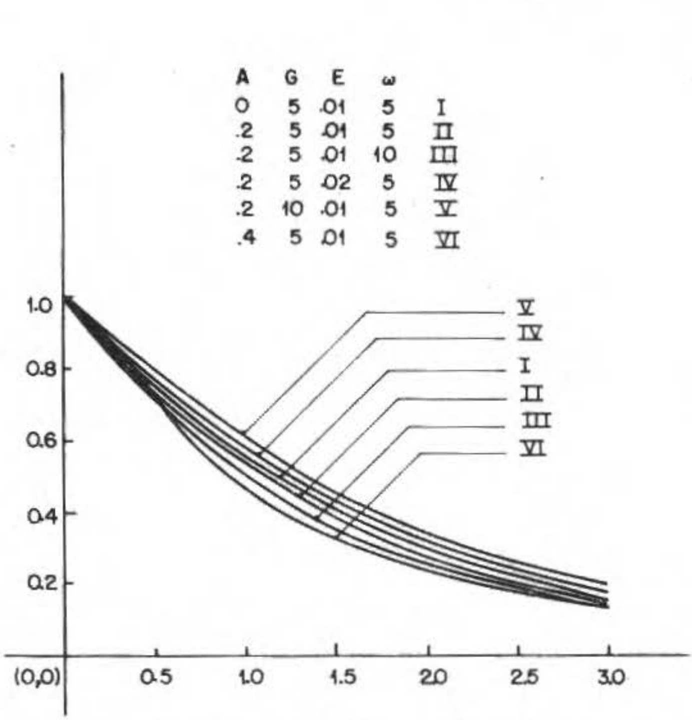

Figure 2. Transient temperature profiles
(ε=0.2, $\omega t=\pi/2$, P=0.72)

where

$$B = B_r + iB_i = \left.\frac{du_1}{dy}\right|_{y=0} \quad \text{and} \quad \tan\alpha = B_i/B_r$$

The numerical values of $|B|$ and tan α are entered in Table 1. We observe from this table that the amplitude of the skin-friction still decreases with increasing the frequency ω even in the presence of variable suction, $A \neq 0$. But an increase in the suction parameter A or the Grashof number G or due to greater viscous dissipative heat, there is an increase in the value of the amplitude $|B|$. For $A \neq 0$, the values of tan α being negative, we conclude that there is always a phase-lag.

From the temperature field, we can now study the amplitude and the phase of the rate of heat transfer. It is given by

$$q' = -k \left. \frac{\partial T'}{\partial y'} \right|_{y'=0} \tag{28}$$

Table 1. Values of $|B|$ and $\tan \alpha$, $|Q|$ and $\tan \beta_1$

A	G	E	ω	$\|B\|$	$\tan \alpha$	$\|Q\|$	$\tan \beta_1$
0	5	0.01	5	2.5260	−0.94108	1.2483	0.63667
			10	1.7722	−0.98718	1.6229	0.70725
			15	1.4400	−1.00130	1.9142	0.74762
.2	5	0.01	5	8.4393	−0.80514	1.2709	0.56568
			10	6.3066	−0.83112	1.6366	0.66321
			15	5.2743	−0.85102	1.9244	0.71466
.2	5	0.02	5	8.4749	−0.80317	1.2649	0.62671
			10	6.3441	−0.82835	1.6382	0.69523
			15	5.3091	−0.84851	1.9272	0.73592
.2	10	0.01	5	16.9630	−0.80304	1.2625	0.73848
			10	12.7200	−0.82572	1.6440	0.75041
			15	10.6630	−0.84588	1.9338	0.77174
.4	5	0.01	5	14.3610	−0.78302	1.2967	0.50119
			10	10.8480	−0.80780	1.6517	0.62160
			15	9.1140	−0.82822	1.9354	0.68305

From (6) and (28), we have

$$q = -\frac{q' \nu}{v'_o k(T_w - T_\infty)} = \left. \frac{d\theta_o}{dy} \right|_{y=0} + \varepsilon\, e^{i\omega t} \left. \frac{d\theta_1}{dy} \right|_{y=0} \tag{29}$$

The mean rate of heat transfer has already been discussed in Soundalgekar [11]. The rate of heat transfer can be written in terms of the amplitude and phase as

$$q = \left. \frac{d\theta_o}{dy} \right|_{y=0} + \varepsilon\, |Q| \cos(\omega t + \beta_1) \tag{30}$$

where

$$Q = Q_r + iQ_i = \left. \frac{d\theta_1}{dy} \right|_{y=0} \quad \text{and} \quad \tan \beta_1 = \frac{Q_i}{Q_r}$$

The numerical values of $|Q|$ and tan β_1 are entered in Table 1. We observe from this table that the amplitude $|Q|$ increases with increasing the frequency ω for all the values of the suction parameter A. An increase in A or due to greater viscous dissipative heat, there is an increase in the value of the amplitude $|Q|$. The values of tan β_1 being observed to be positive, we conclude that there is always a phase-lead in case of the rate of heat transfer.

CONCLUSIONS

1) An increase in A leads to a decrease in the transient velocity and temperature, but it decreases with increasing G or due to greater viscous dissipative heat.

2) The amplitude of the skin-friction $|B|$ increases with increasing the suction parameter A or the Grashof number G or due to greater viscous dissipative heat but decreases with increasing the frequency ω.

3) There is always a phase-lag in case of the skin-friction.

4) The amplitude of the rate of heat transfer $|Q|$ increases with increasing the suction parameter A or the frequency ω or due to greater viscous dissipative heat or due to increasing G.

5) There is always a phase-lead in case of the rate of heat transfer.

REFERENCES

[1] F.K. Moore - Unsteady laminar boundary layer flow, NACA TN 2471, 1951.

[2] M.J. Lighthill - The response of laminar skin-friction and heat transfer to fluctuations in the stream velocity, Proc. Roy. Soc. (London), 224A, 1954, 1.

[3] C.C. Lin - Motion in the boundary layer with a rapidly oscillating external flow, Proc. 9th Int. Congress Appl. Mech., Brussels, 4, 1957, 155.

[4] P.G. Hill and A.H. Stenning Laminar boundary layers in oscillatory flow, J. Basic Engineering, Trans. ASME, Series D, 82, 1960, 593.

[5] R.J. Nickerson - The effect of free-stream oscillation on the laminar boundary layers on a flat plate, Ph.D. Thesis, at M.I.T., Dept. of Mech. Engng. Also WADC TR 57-481, ASTIA Dec. Nº AD 210477, 1958.

[6] J.T. Stuart - A solution of the Navier-Stokes and energy equations illustrating the response of skin-friction and temperature of an infinite plate thermometer to fluctuations in the stream velocity, Proc. Roy. Soc. (London), 231A, 1955, 116.

[7] S.A.S. Messiha - Laminar boundary layers in oscillatory flow along an infinite flat plate with variable suction, Proc. Camb. Phil. Soc., 62, 1966, 329.

[8] V.M. Soundalgekar - Free convection effects on oscillatory flow of a viscous incompressible fluid past an infinite vertical plate with constant suction, Proc. Royal Soc. (London), vol. A333, 1973, 25.

[9] V.M. Soundalgekar - Free convection effects on oscillatory flow of a viscous incompressible fluid past an infinite vertical porous plate with constant suction, Proc. Royal Soc. (London), vol. A333, 1973, 37.

[10] V.M. Soundalgekar - Free convection effects on oscillatory flow of a viscous, incompressible fluid past an infinite plate with variable suction, 5th Int. Heat Transfer Conf., Tokyo, 1974, 373.

[11] V.M. Soundalgekar - Unsteady forced and free convective flow past an infinite vertical porous plate with oscillatory wall temperature and constant suction, Jr. Astroph. Space Sci., 66, 1979, 223.

INVERSE HEAT CONDUCTION PROBLEMS

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ABSTRACT

A whole domain function specification approach and a splitting-up procedure are utilized to develop a computationally efficient method for solving linear inverse heat conduction problems subjected to time variation of temperature, heat flux or ambient temperature at only one of the boundary surfaces of the region. The use of segmented polynomials in the time variable to represent the applied surface condition is found sufficiently versatile to accommodate a wide variety of surface conditions involving smooth variations as well as an abrupt change in the surface temperature, heat flux or ambient temperature. A methodology is presented to estimate qualitatively the stochastic error in the predicted surface condition before measurements are taken. The confidence bounds provide information not only on the precision of the estimated temperatures in the inverse analysis, but also on whether any corrective action is needed to improve the precision of predictions.

NOMENCLATURE

Bi	Biot number
$F(t)$	boundary condition function
n	total number of measured interior temperatures
n_1	number of measured temperatures on or before the abrupt change time, τ
n_2	number of measured temperatures after the abrupt change time, τ
r	radial space variable

Palestra convidada, apresentada pelo Prof. Özisik no 2º Encontro Nacional de Ciências Térmicas - - ENCIT 88, em Águas de Lindóia, SP, Dezembro 1988.

t, t_i	time; time at the measured ith temperature
t_{max}	the duration of measurements
T(r,t)	temperature
T_i	measured ith temperature

Greek Letters

$\theta_{i,j}, \theta_{2,j}$	coefficients defined by Eq. (2)
ϵ_i	measurement errors
$\sigma_{\hat{F}}^2$	variance of the surface condition, F(t)
σ_T^2	variance of the measured interior temperatures, T_i
τ	time at which an abrupt change occurs in the surface condition, F(t)

Subscripts

1	time interval on or before the abrupt change time, τ
2	time interval after the abrupt change time, τ
I	contribution from the initial temperature
j	contribution from the jth term representing the surface condition

Superscripts

^ estimated parameter

INTRODUCTION AND LITERATURE SURVEY

Direct heat conduction problems are concerned with determining the interior temperature distribution of a solid from a known partial differential equation, thermophysical properties, internal energy source, initial temperature and boundary conditions. Conversely, inverse heat conduction problems (IHCP) involve determining unknown thermal properties, internal source, initial temperature and/or boundary conditions from a known partial differential equation and the remaining known initial, boundary, etc. conditions plus some incomplete information about the interior temperature history and/or overspecified boundary conditions. The many types of inverse heat conduction problems all share the characteristic of being mathematically ill-posed in that the inverse solution is sensitive to small changes in the input data in its initial formulation. Examples of inverse heat conduction problems include the estimation of surface temperature, heat flux or heat transfer coefficient at one of the boundaries of a region from discrete interior temperature measurements and known boundary conditions at the other boundaries; determination of thermal conductivity and diffusivity from temperature measurements taken in the interior region and at the boundaries; estimation of the solid-liquid interface position in a melting or freezing substance from temperature data taken in the solid region.

Practical applications of inverse heat conduction analysis are numerous. Frequently the direct measurement of surface temperature is not possible so that the surface conditions must be inferred from the interior temperature distribution. For instance, measurement of surface temperature inside a gun barrel or of an internal combustion engine cylinder sidewall can not be done by direct thermocouple measurement without damaging the thermocouple. Harsh chemicals or very high temperatures at a boundary may also damage a surface mounted temperature sensor. In other situations, the presence of a thermocouple changes the actual boundary conditions. The heat transfer between contacting surfaces is strongly dependent on the small scale surface roughness and geometry; the presence of a thermocouple on the surface alters the surface properties being measured. If the surface is subjected to thermal radiation, presence of the temperature sensor may disturb the surface radiative properties such as emissivity and reflectivity. In many applications the boundary of interest is moving due to ablation or phase change making direct surface measurements impractical. These examples all concern the IHCP problem of determining an unknown surface condition (temperature, heat flux, heat transfer coefficient) from measurements taken below the surface. Measurement of constant or temperature dependent thermal properties also involves the solution of an IHCP. In casting and welding processes, a specific solid-liquid interface position with time is frequently desired. The manner in which cast metals solidify affects the strength of the casting. The shape of the weld pool affects the weld strength. Hence, the capability of determining the required external boundary conditions and initial temperature distribution needed to produce a particular solid-liquid interface in the interior region through the solution of an inverse problem is very useful.

The familiar direct heat conduction problems are mathematically well-posed in the Hadamard sense [1, pp. 7- 9]; that is, a solution exists, the solution is unique and the solution is stable. Stability is taken to mean small variations in the known input information produce small variations in the problem solution. Conversely, inverse heat conduction problems are ill-posed in their initial formulation. An ill-posed problem does not yield a solution satisfying all the above conditions in its present formulation. Gradually, researchers realized that many real physical problems are truly improperly posed. Recently, Tikhonov and Arsenin [1], Payne [2] and Morozov [3] have presented detailed mathematical analysis of improperly posed problems and formalized our understanding of inverse problems in general. Beck, Blackwell and St. Clair, Jr. discuss the solution of inverse heat conduction problems in particular [4] and present an extensive review of the subject. An extensive bibliography of the Soviet research on IHCP is given in the references [1,3]. Many types of IHCP problems have appeared in the literature including determination of surface conditions [1,4,6,7], measurement of thermophysical properties [8,9], inverse phase change problems [10,11], estimation of an initial temperature distribution [12] and determination of the geometry of the solid region [13] to name a few.

Since the solution of an equivalent direct heat conduction problem is implicit in the inverse heat conduction solution, it is not surprising that virtually all of the direct heat conduction techniques appear in the IHCP literature. For linear problems various eigenfunction expansion techniques have been used including Duhamel's theorem (or convolution integral), the integral transform approach and Laplace transforms [6, 14-16]. These approaches naturally lead to the solution of Fredholm and Volterra-type integral equations. For nonlinear problems various finite difference and finite element approaches have been used in the solution of IHCP problems [17-23]. Other papers present more computationally efficient versions of solving the direct problem, such as, methods utilizing rapidly converging eigenfunction series [14,15,16] and linearized problems [24]. For periodic problems trigonometric functions are frequently used [27,28], but later work uses a polynomial periodical B-spline basis for such problems [25,29]. By restricting the allowable functional form of the inverse quantity to a representation with a few parameters, the inverse problem becomes a discrete parameter estimation problem instead of a function estimation problem. Finite difference and finite element methods can be used to discretize the inverse quantity too [18-23].

Probably the most important aspect of the inverse solution is the method of stabilization of smoothing used. Frequently used is an inverse parameter set with a lower dimension than the number of independent input data [4,6,29]. Another approach is to augment the minimization function with terms which penalize large values, slopes and/or curvature. This method is known as the regularization method [1,4]. A combination of these two approaches can be used [31]. Other workers introduce smoothing from the outset of the problem by modelling the parabolic heat conduction with the hyperbolic heat conduction equation [21,32]. Digital filters have been used to presmooth the input data or smooth the unstabilized results [18]. Various methods advanced in linear algebra can be used such as damped least squares and singular-value decomposition.

So far only deterministic aspects have been considered. Since practical inverse heat conduction problems involve measurement errors, some researchers have included a statistical analysis of the effects of measurement error on the computed inverse results [4,18,29,33-35].

An important aspect of inverse heat conduction research concerns optimal inverse results. Some researchers have recognized the importance of computationally efficient algorithms [14,24,25], as previously mentioned. The main concern is optimizing the amount of smoothing in the inverse solution. The more parameters or flexibility included in the analysis (not more than the number of independent data) and the less the amount of smoothing, the more capable the method is of modelling or determining the true variation. The deterministic error is lowest with the maximal number of parameters and least smoothing. Conversely, the fewer the number of parameters and the more smoothing included, the less sensitive the parameters are to the measurement errors. The stochastic error decreases with the number of parameters and amount of smoothing. The optimal amount of smoothing can be estimated statistically. Several papers include statistical methods for matching the level of smoothing to the level of error in the input data [1,4,26,30,35].

Determination of thermophysical properties is another active area in IHCP presently [8,9,30,40-60], particularly with respect to Soviet research. Early works emphasize improved or alternate methods of minimizing the error function between the measured and estimated data. The methods utilized include steepest descent [30,49,9], exhaustive search [40], semi-trial and error [41], Newton [43], random search [44], conjugate gradients [8,45,49,51,54,59,60] and bisection [53]. Some papers treat special problems such as composite media [59], particular materials [9,51], porous media [54] and phase change problems [61]. More recent studies consider attainment of optimal results and experiment planning [55,56,58].

WHOLE DOMAIN FUNCTION SPECIFICATION METHOD FOR LINEAR INVERSE HEAT CONDUCTION

The whole domain function specification approach can be applied to the inverse problem of computing the surface temperature and heat flux at one surface based on known boundary conditions at the other surfaces and discrete measurements of the interior temperature. The whole time domain method makes use of a splitting-up procedure in solving the equivalent direct problem to obtain fast converging series expansions. The method can accommodate discontinuous variations in surface temperature and the statistical analysis gives the effects of measurement errors on the inverse results. The deterministic part of the methodology in a generalized notation is given by Flach and Özişik [35] and a one-dimensional analysis is presented by Al-Najem and Özişik [14]. Here we consider a one-dimensional solid cylinder with no energy generation and zero initial condition, but subjected to an unknown, time-varying convection boundary condition. The appropriate boundary value problem is given by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T(r,t)}{\partial r}\right) = \frac{\partial T(r,t)}{\partial t} \qquad 0 \le r \le 1 \qquad (1a)$$

$$\frac{1}{Bi}\frac{\partial T}{\partial r} + T = F(t) \qquad r = 1 \qquad (1b)$$

$$T(r,0) = 0 \qquad t = 0 \qquad (1c)$$

where F(t) is the time-varying ambient temperature to be estimated by the inverse analysis. We choose the functional form for F(t) over both phases as a quadratic polynomial in t given by

$$F(t) = \sum_{j=0}^{2} \theta_{1j} t^j \qquad t \leq \tau \tag{2a}$$

and

$$F(t) = \sum_{j=0}^{2} \theta_{1j} \tau^j + \sum_{j=0}^{2} \theta_{2j} (t-\tau)^j \qquad t > \tau \tag{2b}$$

The solution to problem (1) is readily obtained using the splitting-up procedure and the resulting solution can be arranged in the form

$$T(r,t) = \sum_{j=0}^{2} \theta_{1j} X_{1j}(r,t) \tag{3a}$$

and

$$T(r,t) = \sum_{j=0}^{2} \theta_{1j} X_{2j}(r,t) + \sum_{j=0}^{2} \theta_{2j} X_{3j}(r,t) \tag{3b}$$

where the X functions are listed in reference [35].

Having established the formalism for the solution of the direct problem (1), we now examine the solution for the inverse problem which is concerned with the determination of the unknown applied surface temperature F(t) from the temperature measurements taken at an interior point of the body.

Suppose n temperature measurements are taken at a location $\mathbf{x}_1$ at times t_i $(i = 1, 2, \ldots, n)$. Let the first n_1 of which be taken on or before the time $t = \tau$ and the remaining $n_2 = n - n_1$ taken after $t = \tau$. Here the time τ, at which an abrupt change occurs in the surface condition, is an unknown; therefore the parameters n_1 and n_2 are also unknowns. Assuming that the measurements are subject to errors ϵ_i with zero means, the measured temperatures, T_i are related to the formal solution, given by equations (3), as

$$T_i = \sum_{j=0}^{2} \theta_{1j} X_{1j}(\mathbf{x}_1, t_1) + \epsilon_{1i} \qquad i = 1, 2, \ldots, n_1 \tag{4a}$$

and

$$T_i = \sum_{j=0}^{2} \theta_{1j} X_{2j}(\mathbf{x}_1, t_i) + \sum_{j=0}^{2} \theta_{2j} X_{3j}(\mathbf{x}_1, t_i) + \epsilon_{2i} \quad i = (n_1+1), (n_1+2), \ldots, n \tag{4b}$$

with

$$t_{n1} \leq \tau < t_{n_1+1} \tag{4c}$$

Clearly, due to the presence of random measurement errors, ϵ_i, the measured temperatures, T_i, are also random variables.

The solution of the inverse problem as defined by equations (4) is now reduced to that of estimating the unknown parameters θ_{1j}, θ_{2j}, τ and n_1, which is essentially a problem in two-phase regression. Estimates to the true values of the unknown parameters θ_{1j}, θ_{2j}, τ and n_1 can be obtained without making further assumptions about the distribution of the errors, ϵ_i, by using the least squares technique as now described.

The residual sum of squares (SSE) over both phases of the regression is obtained from equation (4) as

$$\begin{aligned} SSE &= \sum_{i=1}^{\hat{n}_1} \hat{\epsilon}_{1i}^2 + \sum_{i=\hat{n}_1+1}^{n} \hat{\epsilon}_{2i}^2 \\ &= \sum_{i=1}^{\hat{n}_1} \left[T_i - \sum_{j=0}^{2} \hat{\theta}_{1j} X_{1j} \right]^2 \\ &+ \sum_{i=\hat{n}_1+1} \left[T_i - \sum_{j=0}^{2} \hat{\theta}_{1j} X_{2j} - \sum_{j=0}^{2} \hat{\theta}_{2j} X_{3j} \right]^2 \end{aligned} \tag{5}$$

Here we used the notation $\hat{\theta}_{1j}, \hat{\theta}_{2j}$ and $\hat{n}_1$ to denote estimates of the true values of θ_{1j}, θ_{2j} and n_1, respectively. The SSE is minimized with respect to the linearly occuring parameters $\hat{\theta}_{1j}$ and $\hat{\theta}_{2j}$ by differentiating equation (5) with respect to $\hat{\theta}_{1j}$ and $\hat{\theta}_{2j}$ and setting these derivatives equal to zero.

To minimize with respect to the discrete unknown $\hat{n}$, we assume that measurements are taken with sufficiently small intervals that negligible error occurs by setting $\hat{\tau} = t_{\hat{n}1}$. Then, for a specified value of $\hat{n}_1$ (which now also fixes $\hat{\tau}$), differentiating equation (5) with respect to $\hat{\theta}_{1j}$ and $\hat{\theta}_{2j}$ and setting the derivatives equal to zero, we obtain a set of equations for each possible value of $\hat{n}_1$ (or equivalently $\hat{\tau}$).

The unknowns $\hat{\theta}_{1j}$ and $\hat{\theta}_{2j}$, for each possible value of $\hat{n}_1$ (or $\hat{\tau}_1$) is determined from the solution of these equations, for each possible value of $\hat{n}_1$ (or $\hat{\tau}$). Clearly, the $\hat{n}_1$ (equivalently $\hat{\tau}$) which produces the minimum SSE is the estimated value of the true value n_1 (or τ).

CONFIDENCE BOUNDS

Sometimes lacking in the IHCP literature is the statistical analysis of the effects of measurement errors on the estimated boundary condition. Confidence limits on the inverse solution are a crucial aspect of the IHCP. Without the confidence bounds on the estimated surface condition, the validity and accuracy of the inverse solution is in doubt. Approximate confidence bounds are now developed for the estimated boundary condition.

The difficulty in the statistical analysis comes from the discrete parameter n_1 and nonlinear parameter τ. In the present work we neglect any errors in estimating n_1 and τ and develop approximate statistical inferences concerning the stochastic error in the estimated surface condition. We also assume uncorrelated, normally distributed measurement errors with constant variance and zero mean. Then approximate statistical inferences concerning the estimated parameters $\hat{\theta}_{1j}$ and $\hat{\theta}_{2j}$ are readily obtained. Under these assumptions, the estimated parameters, $\hat{\theta}_{1j}$ and $\hat{\theta}_{2j}$, are unbiased, and the variance-covariance matrix of the $\hat{\theta}_{1j}$ and $\hat{\theta}_{2j}$ coefficients is determined. The variance of the estimated surface condition (i.e., $\sigma_{\hat{F}}^2$) depends on the precision of the measurements σ_T, the measurement location $\mathbf{x}_1$, the times of the measurements t_i, the total number of measurements n, the time of the abrupt change in the boundary condition $\hat{\tau}$ (or $\hat{n}_1$) and the sensitivity coefficients X, while X depends only on the geometry and the boundary conditions for the problem considered. Therefore, for a specific application selected from the general problem given by equations (1), the sensitivity coefficients are fixed. Then for a particular problem, var $(\hat{F})$ depends on $\sigma_t, \mathbf{x}_1, t_i, \hat{\tau}$ (or $\hat{n}_1$) and n.

Once an estimate $\hat{\tau}$ is obtained from the measured data, approximate confidence bounds are for the estimated surface condition $\hat{F}(t)$. For example 99% confidence bounds on $\hat{F}(t)$ are

$$\hat{F}(t) \pm 2.576\sigma_{\hat{F}}(t) \tag{6}$$

since 99% of a normally distributed population is contained within ±2.576 standard deviations of the mean. Since the variance in $\hat{\tau}$ (or $\hat{n}_1$) has been neglected, the true confidence level associated with equation (6) is lower than 99%.

RESULTS AND DISCUSSION

To illustrate the application of the foregoing inverse analysis in the determination of the surface condition and the confidence bounds, we consider a one-dimensional, solid cylinder with no energy generation and zero initial temperature and a time varying surface temperature $F(t)$. We assume an abrupt change in the applied surface temperature $F(t)$ at a time τ.

We examine, first, the effects of parameters such as the measurement location $\mathbf{x}_1 = r_1$, the number of measurements n, etc., on the normalized standard deviation $\sigma_{\hat{F}}/\sigma_T$ of the estimated boundary temperature. Figures 1a, b, c, d illustrate the effects of, respectively, the measurement location r_1, the number of measurements n, the time of the abrupt change in the surface temperature $\hat{\tau}$, and the length of the time interval over which measurements are taken, t_{max}, on the normalized standard deviation for the cylinder problem described above for evenly spaced data measurements and $Bi = \infty$ (i.e., prescribed surface temperature). Figure 1a shows, as expected, that the standard deviation decreases as the measurement location r_1 is moved towards the surface subjected to the unknown temperature variation. Increasing the number of measurements, n, taken over the total measurement time t_{max} decreases the standard deviation as shown in Fig. 1b. The time $\hat{\tau}$ at which an abrupt change occurs in the surface temperature strongly affects the standard deviation as illustrated in Fig. 1c. Clearly, peaks occur in the standard deviation at times $\hat{\tau}$ when an abrupt change occurs in the temperature. Note that the peaks shown in Figs. 1a and 1b coincide with the location of $\hat{\tau}$. Figure 1d shows that the shorter the total duration of measurements, t_{max}, the larger the standard deviation because less information is reaching the measurement location. Standard deviation charts such as those illustrated in

Figs. 1a-d are quite useful in planning the optimal conditions for measurements in inverse studies.

We now utilize the present inverse method to analyze simulated measured data for the case $Bi = \infty$ (prescribed surface temperature). Figure 2a shows the simulated measured data produced by adding pseudo-normal errors with zero means and $\sigma_T = 0.01$ to the exact centerline temperature for a step change in surface temperature. Figure 2b shows the exact surface temperature $F(t)$ and our predictions $\hat{F}(t)$ with the present method of inverse analysis. Included on this figure are the 99% confidence bounds on $\hat{F}(t)$. Indeed, the exact temperature lies within the confidence bounds. In these computations, we consider the temperature probe located at the centerline, $r_1 = 0.100$, evenly-spaced measurements are the time interval $0 < t \leq 1$ and the precision of the measurements is $\sigma_T = 0.01$.

Figures 3a,b are prepared to illustrate the usefulness of the confidence bounds in determining whether sufficient measured information has been obtained to accurately estimate the unknown surface temperature. In Figure 3a the broad confidence bounds near the end of the measurements suggest that insufficient information has been received to adequately predict the surface temperature in this region. Indeed, the estimated surface temperature is significantly in error near the end of measurements. Insufficient measurements may have been taken to properly estimate the temperature over the entire time domain. Therefore, we extend the length of time over which measurements are taken from $t_{max} = 1$ to $t_{max} = 1.5$ as shown in Fig. 3b. Now the confidence bounds indicate a well estimated surface temperature over the entire time domain which is in agreement with the actual predicted temperature. Hence, the confidence bounds not only establish the precision of the estimated temperature, but also provide information whether corrective action is needed to obtain a specified level of precision.

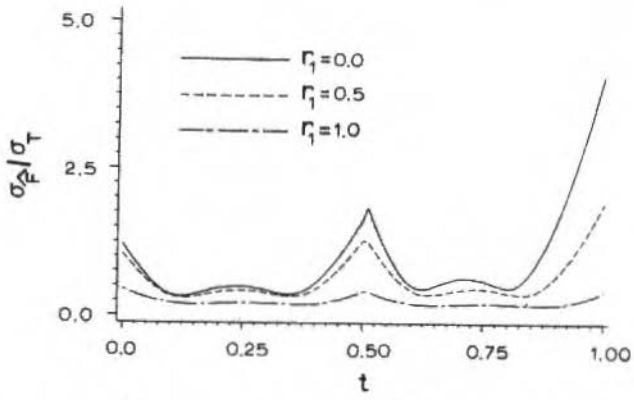

Figure 1a. Effect of measurement location, r_1, on the normalized standard deviation, $\sigma_{\hat{F}}/\sigma_T$, of the estimated surface temperature ($n = 100$, $\hat{r} = 0.5$, $t_{max} = 1$).

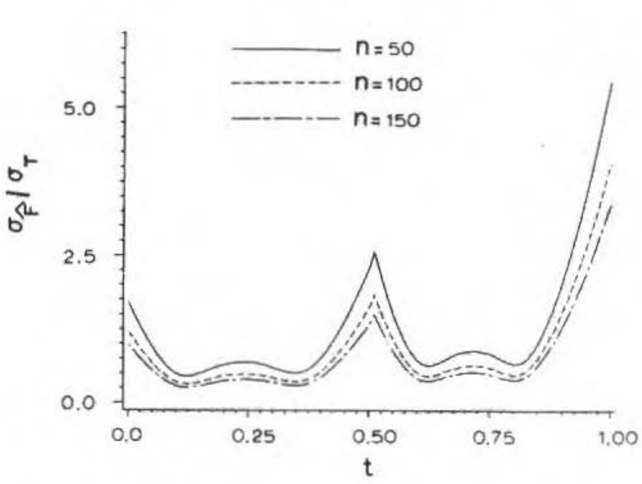

Figure 1b. Effect of the number of measurements, n, on the normalized standard deviation, $\sigma_{\hat{F}}/\sigma_T$, of the estimated surface temperature ($r_1 = 0, \hat{r} = 0.5, t_{max} = 1$).

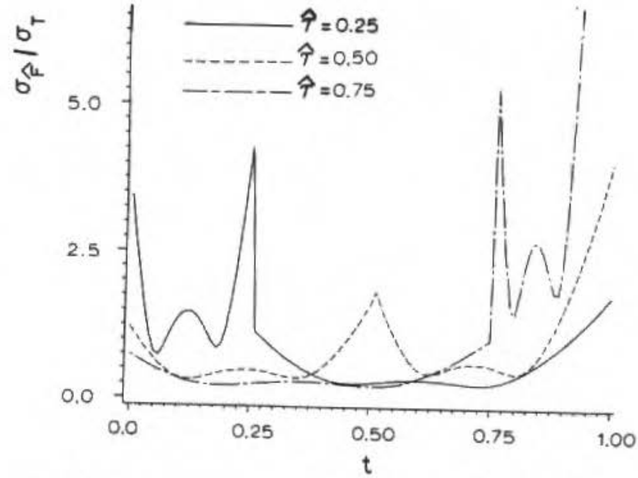

Figure 1c. Effect of the time of the abrupt change in the surface temperature, $\hat{\tau}$, on the normalized standard deviation, $\sigma_{\hat{F}}/\sigma_T$, of the estimated surface temperature $(r_1 = 0, n = 100, t_{max} = 1)$.

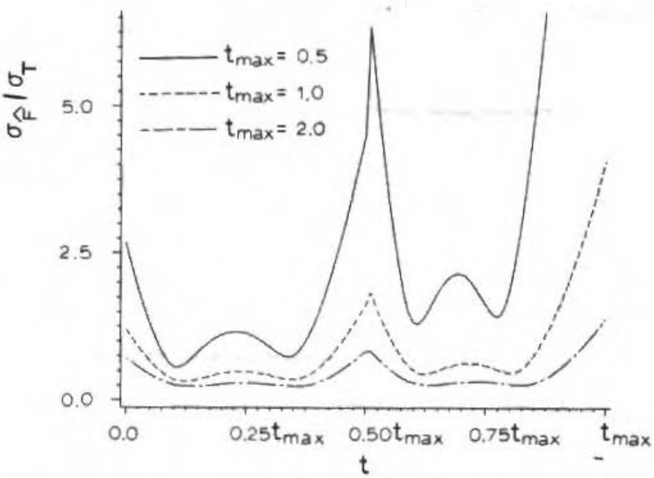

Figure 1d. Effect of the duration of meaurements, t_{max}, on the normalized standard deviation, $\sigma_{\hat{F}}/\sigma_T$, of the estimated surface temperature $(r_1 = 0, n = 100, \hat{\tau} = 0.5t_{max})$.

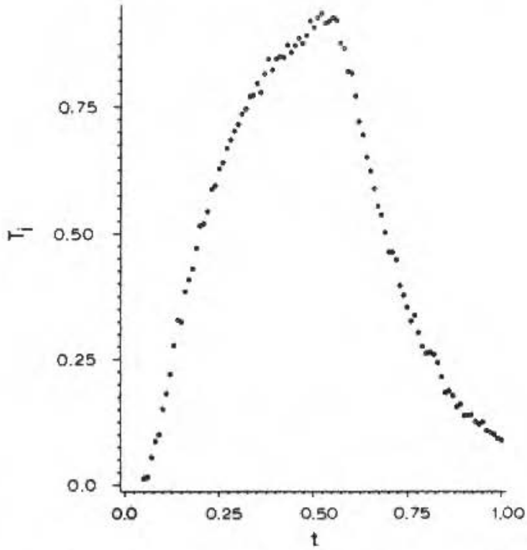

Figure 2a. Simulated measured centerline temperature, T_i, as a function of time, t, for a step change in surface temperature as shown in Figure 2b.

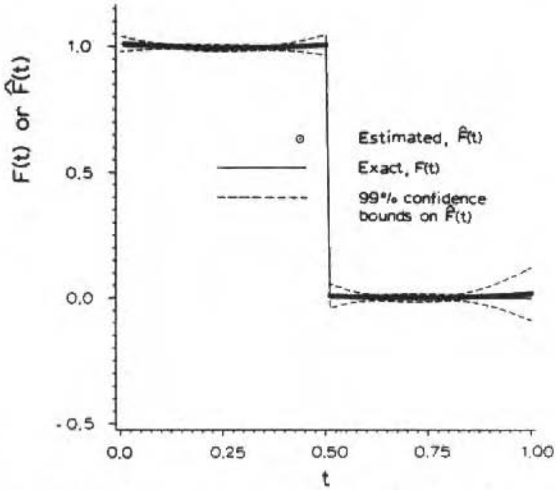

Figure 2b. Estimated surface temperature, $\hat{F}(t)$, and the 99% confidence bounds for a step change in the surface temperature $(r_1 = 0, n = 100, t_{max} = 1)$.

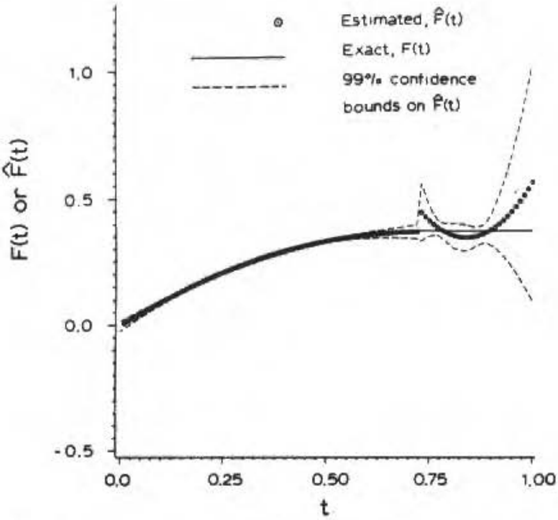

Figure 3a. An illustration of the effects of insufficient measurements on the estimated surface temperature after an abrupt change $(r_1 = 0, n = 100, t_{max} = 1)$.

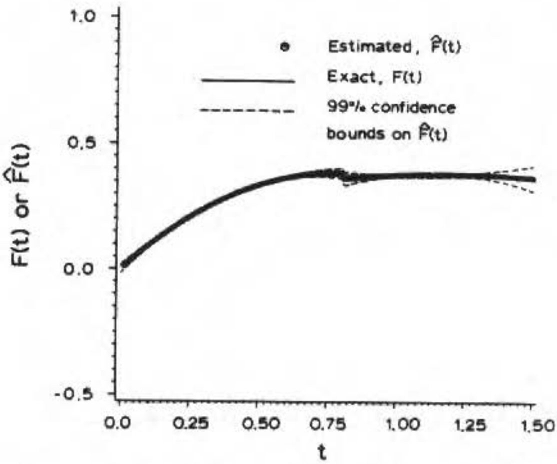

Figure 3b. An illustration of overcoming the difficulty shown in Figure 5a by taking additional measurements $(r_1 = 0, n = 150, t_{max} = 1.5)$.

REFERENCES

[1] Tikhonov, A. N. and Arsenin, V. Y. - Solutions of ill-posed problems, Winston and Sons, Washington, DC, 1977.

[2] Payne, L. E. - Improperly posed problems in partial differential equations, SIAM, Philadelphia, PA, 1975.

[3] Morozov, V. A. - Methods of solving incorrectly posed problems, Springer-Verlag, New York, 1984.

[4] Beck, J. V.; Blackwell, B. and St. Clair, Jr., C. R. - Inverse heat conduction, Wiley, New York, 1985.

[5] Burggraf, O. R. - An exact solution of the inverse problem in heat conduction theory and applications, J. Heat Transfer, 86C, p.373-382, 1964.

[6] Beck, J. V. - Calculation of surface heat flux from an internal temperature history, ASME Paper 62-HT-46, 1962.

[7] Frank, I. - An applicaiton of least squares method to the solution of the inverse problem of heat conduction, J. Heat Transfer, p.378-379, 1963.

[8] Artyukhin, E. A. - Recovery of the temperature dependence of the thermal conductivity coefficient from the solution of the inverse problem, J. Eng. Phys., 19, p.698-702, 1982.

[9] Yarny, Y.; Delaunay, D. and Bransier, J. - Identification of nonlinear thermal properties by an output least square method, 8th Int. Heat Transfer Conference, p.1811-1816, 1986.

[10] Katz, M. A. and Rubinsky, B. - An inverse finite element technique to determine the change of phase interface location in one-dimensional melting problems, Numer. Heat Transfer, 7, p.269-283, 1984.

[11] Reemtsen, R. and Kirsch, A. - A method for the numerical solution of the one-dimensional inverse Stefan problem, Numer. Math., 45, p.253-273, 1984.

[12] Nikitenko, N. I. and Postil, S. D. - Numerical solution of retrospective heat-conduction problems, J. Eng. Phys., 45, p.1259-1263, 1984.

[13] Ksieh, C. K. and Kassab, A. J. - A general method for the solution of inverse heat conduction problems with partially unknown system geometries, Int. J. Heat Mass Transfer, 29, p.47-58, 1986.

[14] Alnajem, N. M. and Ozisik, M. N. - A direct analytical approach for solving linear inverse heat conduction problems, J. Heat Transfer, 107, p.700-703, 1985.

[15] Alnajem, N. M. and Ozisik, M. N. - Inverse heat conduction in composite layers, ASME National Heat Transfer Conference, Denver, Colorado, Aug. 1985, ASME Paper #85-HT-53, 1985.

[16] Alnajem, N. M. and Ozisik, M. N. - On the solution of three-dimensional inverse heat conduction in finite media, Int. J. Heat Mass Transfer, 28, p.2121-2128, 1985.

[17] Beck, J. V. - Nonlinear estimation applied to the nonlinear inverse heat conduction problem, Int. J. Heat Mass Transfer, 13, p.703-716, 1970.

[18] Hensel, E. and Hills, R. G. - An initial value approach to the inverse heat conduction problem, J. Heat Transfer, 108, p.248-256, 1986.

[19] Raynaud, M. and Bransier, J. - A new finite-difference method for the nonlinear inverse heat conduction problem, Numer. Heat Transfer, 9, p.27-42, 1986.

[20] D'Souza, N. - Numerical solution of one-dimensional inverse transient heat conduction by finite difference method, ASME Paper 75-WA/HT-81, 1975.

[21] Weber, C. F. - Analysis and solution of the ill-posed inverse heat conduction problem, Int. J. Heat Mass Transfer, 24, p.1783-1792, 1981.

[22] Krutz, G. W.; Schoenhals, R. J. and Hore, P. S. - Application of the finite-element method to the inverse heat conduction problem, Numer. Heat Transfer, 1, p.489-498, 1978.

[23] Bass, B. R. - Applications of the finite element to the inverse heat conduction problem using Beck's second method, J. Eng. Ind., 102, p.168-176, 1980.

[24] Beck, J. V.; Lithkouhi, B. and St. Clair, Jr., C. R. - Efficient sequential solution of the nonlinear inverse heat conduction problem, Numer. Heat Transfer, 5, p.275-286, 1982.

[25] Flach, G. P. and Ozisik, M. N. - Inverse heat conduction problem of periodically contacting surfaces, J. Heat Transfer, (in press).

[26] Yoskoboinikov, Y. E. and Preobrazhenskii, N. G. - Descriptive solution of the inverse heat-conduction problem in the B-spline basis, J. Eng. Phys., 45, p.1254-1258, 1984.

[27] Overbye, V. D.; Bennethum, J. E.; Uyehara, O. A. and Myers, P. S. - Unsteady heat transfer in engines, SAE Transactions, 69, p.461-494, 1961.

[28] France, D. M. and Chiang, T. - Analytic solution to inverse heat conduction problems with periodicity, J. Heat Transfer, 102, p.579-581, 1980.

[29] Flach, G. P. and Ozisik, M. N. - Periodic B-spline basis for quasi-steady periodic inverse heat conduction, Int. J. Heat Mass Transfer, 30, p.869-880, 1987.

[30] Artyukhin, E. A. - Determination of thermal diffusivity from experimental data, J. Eng. Phys., 29, p.878-881, 1975.

[31] Beck, J. V. and Murio, D. A. - Combined function specification-regularization procedure for solution of inverse heat conduction problem, AIAA-84-0491, 1984.

[32] Novikov, N. A. - Hyperbolic equation of thermal conductivity. Solution of the direct and inverse problems for a semi-infinite bar, J. Eng. Phys., 35, p.1253-1257, 1979.

[33] Murio, D. A. - The mollification method and the numerical solution of an inverse heat conduction problem, SIAM J. Stat. Comput., 2, p.17-34, 1981.

[34] Backus, B. and Gilbert, F. - Uniqueness in the inversion of inaccurate gross earth data, Phil. Trans. R. Soc., Lond., 266, p.123-192, 1970.

[35] Flach, G. P. and Ozisik, M. N. - Whole domain function-specification method for linear inverse heat conduction, Ann. Nucl. Energy, 13, p.325-336, 1986.

[36] Alkidas, A. C. - Heat trasnfer characteristics of a spark-ignition engine, J. Heat Transfer, 102, p.189-193, 1980.

[37] Wendland, D. W. - The effect of periodic pressure and temperature fluctuations on unsteady heat transfer in a closed system, NASA Report CR-72323, 1968.

[38] Updike, W. A. - Computation of surface temperatures for time dependent period convection boundary ocnditions, ASME Paper 84-WA/HT-40, 1984.

[39] Moses, W. - Ph.D. thesis, North Carolina State University, 1986.

[40] Polezhaev, Y. V.; Narozhnyi, Y. G. and Safonov, V. E. - A method for determining the thermal conductivity coefficient of high-temperature materials during their unsteady heating, High Temperature, 11, p.543-548, 1973.

[41] Omel'chenko, K. G. and Pchelkina, V. G. - Solution of an inverse problem of nonlinear heat conduction to determine thermophysical characteristics, J. Eng. Phys., 29, p.885-887, 1976.

[42] Frolov, V. V. - Group properties of the nonlinear heat-conduction equation and the solution of inverse problems, J. Eng. Phys., 30, p.363-369, 1977.

[43] Nikitenko, N. I. and Kolodnyi, Y. M. - Numerical solution of the inverse heat-conduction problem for determining thermal constants, J. Eng. Phys., 33, p.1461-1463, 1978.

[44] Strakhov, V. I.; Leonova, S. I. and Garashchenko, A. N. - The temperature dependence of the thermal constants of composite polymer materials, J. Eng. Phys., 33, p.1454-1457, 1978.

[45] Alifanov, O. M. and Mikhailov, V. V., Solution of the nonlinear inverse thermal conductivity problem by the iteration method, J. Eng. Phys., 35, p.1501-1506, 1979.

[46] Matsevityi, Y. M. and Multanovskii, A. V. - An iterative filter for solution of the inverse heat-conduction problem, J. Eng. Phys., 35, p.1373-1378, 1979.

[47] Matsevityi, Y. M. and Multanovskii, A. V. - Identification of heat-exchange parameters by optimal dynamic filtration, High Temperature, 17, p.867-872, 1980.

[48] Artyukhin, E. A. - Reconstruction of the thermal conductivity coefficient from the solution of the nonlinear inverse problem, J. Eng. Phys., 41, p.1054-1058, 1982.

[49] Goryachev, A. A. and Yudin, V. M. - Solution of the inverse coefficient problem of heat conduction, J. Eng. Phys., 43, p.1148-1154, 1983.

[50] Matsevityi, Y. M. - Identification of thermophysical characteristics by solving inverse heat conductivity problems, J. Eng. Phys., 45, p.1165-1169, 1984.

[51] Artyukhin, E. A.; Killikh, V. E. and Okhapkin, A. S. - Reconstructing the effect coefficient of thermal conductivity of asbestos-textolite from the solution of the inverse problem, J. Eng. Phys., 45, p.1281-1286, 1984.

[52] Abramov, R. T.; Kolesnikov, P. M.; Krejci, L. and Vogel, I. - An inverse method of determining linear variations of thermophysical properties and heat fluxes, Heat Transfer-Soviet Research, 16, p.96-103, 1984.

[53] Iordanov, V. I. and Steward, A. P. - A method and a computer program for determining the thermal diffusivity in a solid slab, Appl. Math. Modelling, 8, p.169-178, 1984.

[54] Tryanin, A. P. - Identification of heat-transfer coefficients in a porous body from the solution of an inverse problem, J. Eng. Phys., 45, p.1301-1305, 1984.

[55] Simbirskii, D. F. and Gulei, A. B. - Optimally planned experimental-computational determination of thermal conductivity of solids in transient heating mode, J. Eng. Phys., 45, p.1227-1231, 1984.

[56] Artyukhin, E. A. and Okhapkin, A. S. - Parametric analysis of solution of a nonlinear inverse problem of recovering the thermal conductivity of a composite material, J. Eng. Phys., 45, p.1275-1280, 1984.

[57] Evseev, V. R.; Mikhailova, L. S. and Nechaev, V. S. - Determining the thermophysical characteristics of materials from the solution of the inverse heat-conduction problem using spline functions, High Temperature, 22, p.562-566, 1985.

[58] Artyukhin, E. A. - Experimental design of measurements for the solution of coefficient-type inverse heat-conduction problems, J. Eng. Phys., 48, p.372-376, 1985.

[59] Artyukhin, E. A. and Okhapkin, A. S. - Determination of the temperature-dependent variation of the thermal conductivity of a composite material from the data of a nonstationary experiment, J. Eng. Phys., 44, p.191-196, 1983.

[60] Gruzdev, V. A. and Kovalenko, Y. A. - Solution of one inverse problem of coefficients for a nonlinear heat conduction equation, Heat Transfer-Soviet Research, 16, p.99-113, 1984.

[61] Tarzia, D. A. - Simultaneous determination of two unknown thermal coefficients through an inverse one-phase Lame-Clapeyron (Stefan) problem with an overspecified condition on the fixed face, Int. J. Heat Mass Transfer, 26, p.1151-1157, 1983.

[62] Larson, K. B. and Koyama, K. - Measurement by the flash method of thermal diffusivity, heat capacity, and thermal conductivity in a two-layer composite samples, J. Appl. Phys., 39, p.4408-4416, 1968.

[63] Touloukian, Y. S.; Powell, R. W.; Ho, C. Y. and Nicolaou, M. C. - Thermophysical properties of matter, volume 10 - thermal diffusivity, 1973.

ON THE DEVELOPMENT OF EFFICIENT ALGORITHMS FOR THREE DIMENSIONAL FLUID FLOW

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ABSTRACT

The difficulties of constructing efficient algorithms for three dimensional flow are discussed. Reasonable choice candidates are analyzed and tested. Most have obvious shortcomings. Yet, there is promise that an efficient class of algorithms exist between the severely time step sized limited explicit or approximately factored algorithms and the computationally intensive direct inversion of large sparse matrices by Gaussian elimination.

INTRODUCTION

The rapid advances in computer hardware and architecture achieved during the last decade have enabled computational fluid dynamicists to solve many complicated two and three dimensional flows. Yet, even with the present computer resources and those expected to be available in the near future, there exist many problems of present engineering interest that remain impractical computations. The prediction of the intensely hot flow about candidate designs for the National Aero-Space Plane (NASP) and through their supersonic combustion ramjet engines are cases in point. New developments in the construction of efficient numerical methods are desperately required now.

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Rapid advances have been made also in computational fluid dynamics. The 1970's saw the development of implicit approximately factored methods using block tridiagonal inversion [1,2] that greatly improved numerical efficiency and enabled computational fluid dynamics to attack a broad range of new complex problems. The emphasis of the 1980's has been on the development of highly accurate procedures that use upwind flux split difference approximations with flux limiters [3-6]. During the mid 1980's the upwind procedures were used to combine implicit block tridiagonal inversion with Gauss-Seidel relaxation to produce efficient unfactored implicit methods [7-10]. The unfactored schemes showed exceptionally fast convergence rates for certain classes of problems in two dimensions. Three dimensional flow solvers have remained persistently slow, however, usually requiring hundreds of integration time steps to converge to steady flow solutions.

Well here we are now on the verge of entering the 1990's. What can we expect from computational fluid dynamics to meet our present large requirements? Can we expect the development highly efficient schemes, those with unbounded integration time step sizes, for compressible viscous flows about or through complex three dimensional geometries? For steady flow solutions, can we approach the efficiency of a Newton procedure for determining the roots of a vector equation? Can the number of iterations required for the solution be reduced to only those necessary because of the nonlinearity of the flow, perhaps a few dozen or so? The present paper discusses some of the difficulties of combining the basic elements of computational fluid dynamics, whose previous development have represented computational triumphs, into efficient methods for three dimensional compressible flow. At present there does not exist such a method nor is it promised that will one be presented herein. It is hoped, however, that by focusing attention on this problem research for its solution will be stimulated.

GOVERNING EQUATIONS AND NUMERICAL METHODS

Although our interest in general concerns both steady and unsteady flows, we will focus our attention on searching for algorithms that converge rapidly toward a steady state. The algorithms under consideration will faithfully approximate the equations governing unsteady flow without trying to fool Mother Nature by preconditioning them or modifying their underlying physics. Nor will strategies for accelerating convergence by tampering with the numerical temporal history of flow evolution,such as local time stepping, be considered. Optimal algorithms for converging rapidly to steady flow should also serve efficiently in solving for time dependent flows. Although viscous effects do not receive attention directly herein, the results of our discussions also apply directly to compressible viscous flow. The numerical solutions to be presented are viscous flow solutions.

We begin our discussion of efficient numerical procedures for three dimensional flows by first considering optimal methods for one and two dimensions.

One Dimension

The unsteady equations governing compressible flow in one dimension can be written in the following vector form.

$$\frac{\partial U}{\partial t} = -\frac{\partial F}{\partial x} \tag{1}$$

The vector U is the state vector whose elements completely specify the flow at a given time at each point in x coordinate space. The vector F is the flux vector that describes the convection of mass, momentum, and energy within the fluid and the stress acting on the fluid.

An implicit finite difference approximation to the above partial differential equation is (see also Fig.1)

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} = -\frac{D}{\Delta x} \cdot F_i^{n+1} = -\frac{F_{i+\frac{1}{2}}^{n+1} - F_{i-\frac{1}{2}}^{n+1}}{\Delta x} \tag{2}$$

where the subscripts refer to a spacial set of mesh points, x_i , spaced Δx apart and the superscripts refer to times t^n and t^{n+1} spaced Δt apart. The symbol D represents a spacial difference operator acting on the flux vector F. The superscript $n+1$ on F implies that its value depends implicitly upon the new value of U at the new time t^{n+1}. Because F is in general a nonlinear function of U, a linearization of F will be required before we can solve the above finite difference equation. For the equations describing compressible flow, the Euler equations, the flux vector can be expressed as

$$F = AU, \qquad \text{where } A = \frac{\partial F}{\partial U}, \qquad \text{the Jacobian of } F \text{ with respect to } U.$$

Using a central difference approximation for D,

$F_{i+\frac{1}{2}} = \dfrac{(A_i U_i + A_{i+1} U_{i+1})}{2}$ where the Jacobian matrices A are evaluated at time t^n.

The difference equation becomes

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{2\Delta x}\left(A_{i+1} U_{i+1}^{n+1} - A_{i-1} U_{i-1}^{n+1}\right)$$

or

$$\hat{B} U_{i+1}^{n+1} + \hat{A} U_i^{n+1} + \hat{C} U_{i-1}^{n+1} = \hat{D} \tag{3}$$

where

$$\hat{B} = \frac{\Delta t A_{i+1}}{2\Delta x}, \qquad \hat{A} = I, \qquad \hat{C} = -\frac{\Delta t A_{i-1}}{2\Delta x}, \text{ and } \hat{D} = U_i^n$$

Eq.(3) represents a block tridiagonal matrix equation, which together with an implicit formulation of the boundary conditions at the end points of the numerical domain, can be solved directly using the Thomas algorithm. This algorithm consists of a forward elimination followed by a backward substitution to directly determine the solution vector U at each grid point.

Alternatively, the flux vector F can be split as follows

$$F_{i+\frac{1}{2}} = A_+ U_i + A_- U_{i+1}$$

where A_+ and A_- are the split Jacobians formed by diagonalizing the matrix A by a similarity transformation, replacing the elements of diagonal matrix containing the eigenvalues of A by two diagonal matrices, one containing only the positive elements and the other containing only the negative elements, and then by re-multiplying by the similarity transformations.

$$A = T^{-1}\Lambda T, \qquad \Lambda = \Lambda_+ + \Lambda_-, \qquad A_+ = T^{-1}\Lambda_+ T, \quad \text{and } A_- = T^{-1}\Lambda_- T.$$

The difference operator D is then used to represent upwind difference approximations for each part of the flux split vector (see Fig.2). There is considerable choice in the present literature on what type of splitting to use and how to determine the split Jacobians from the data at neighboring mesh points near mesh point (i). At present the flux vector difference splitting of Roe [5] appears to be a very good choice for both the solution of the Euler equations and for the compressible Navier-Stokes equations in which the inviscid terms are upwinded.

A simple resulting flux split implicit difference equation has the same form as Eq.(3) but with the following definitions for the block matrix elements.

$$\hat{B} = -\frac{\Delta t}{\Delta x}A_{+_{i+\frac{1}{2}}}, \qquad \hat{A} = I + \frac{\Delta t}{\Delta x}\left(A_{+_{i+\frac{1}{2}}} - A_{-_{i-\frac{1}{2}}}\right), \text{ and } \hat{C} = \frac{\Delta t}{\Delta x}A_{-_{i-\frac{1}{2}}}$$

The flux split formulation is seen from the above to add weight to the diagonal block matrix element $\hat{A}$. This property could be used to solve Eq.(3) indirectly by a relaxation procedure, which will be discussed later.

Eq.(2) can be written in "delta law" form by first defining

$$\delta U_i = U_i^{n+1} - U_i^n$$

and then rewriting Eq.(2) as

$$\left\{I + \Delta t\frac{D}{\Delta x}\cdot A\right\}\delta U_i = -\Delta t\frac{D}{\Delta x}F_i^n = \Delta U_i \tag{4}$$

where the difference operator D always operates on all factors to its right.

The script δU is the change in the solution solved for implicitly by inverting a block tridiagonal matrix equation. The triangular ΔU represents the predicted change in the solution using local data to explicitly approximate the right hand side of Eq.(1), which we will call the driving term of the implicit equation and is also often referred to as the residual term. It contains the local information of the flow physics at each grid point. Although a linearization has been used on the left hand implicit side, the right hand side retains all the nonlinearity of the original partial differential equation. The left hand side of the above equation extends the numerical domain of dependence so that information computed locally can be felt at grid points far away. It globally integrates the data and maintains numerical stability for large time steps.

Eq.(4), solved for each time integration step directly by block tridiagonal inversion, represents an optimal solution procedure for the equations describing one dimensional compressible flow. Solutions should require at most a few dozen time step integrations for convergence. Of course, since we are discussing only one dimensional flow at present there is little need to worry about optimal procedures. Any accurate convenient method, explicit or implicit, should suffice for most one dimensional problems of current engineering interest.

Two Dimensions

The unsteady equations governing compressible flow in two dimensions can be written in the following vector form.

$$\frac{\partial U}{\partial t} = -\frac{\partial F}{\partial x} - \frac{\partial G}{\partial y} \tag{5}$$

An implicit delta law form approximation to Eq.(5) is given by

$$\left\{ I + \Delta t \left(\frac{D}{\Delta x} \cdot A + \frac{D}{\Delta y} \cdot B \right) \right\} \delta U_{i,j} = -\Delta t \left(\frac{D}{\Delta x} F + \frac{D}{\Delta y} G \right)_{i,j}^{n} = \Delta U_{i,j} \tag{6}$$

in which A and B are the Jacobians of the flux vectors F and G with respect to U, respectively.

Upon performing the operations specified by the difference operator D, the following block pentadiagonal matrix equation results.

$$\hat{B}\delta U_{i,j+1} + \hat{A}\delta U_{i,j} + \hat{C}\delta U_{i,j-1} + \hat{D}\delta U_{i+1,j} + \hat{E}\delta U_{i-1,j} = \Delta U_{i,j} \tag{7}$$

The new solution upon solving matrix Eq.(7) is

$$U_{i,j}^{n+1} = U_{i,j}^{n} + \delta U_{i,j}$$

Unfortunately there exists no direct efficient solution of Eq.(7). If the five diagonals were adjacent, or nearly so, then efficient solution would be possible. However, on an $n \times n$ mesh two of the diagonals lie n positions away from the main diagonal,precluding direct efficient inversion. Direct solution is still possible by Gaussian elimination which can be made less costly by taking advantage of the sparseness of the matrix equation. Two other choices are to solve indirectly by Gauss-Seidel line relaxation, which we will discuss later, or directly by approximate factorization, which we discuss now.

Eq.(6) is replaced by the following factored implicit equation.

$$\left\{I + \Delta t \frac{D}{\Delta x} \cdot A\right\} \left\{I + \Delta t \frac{D}{\Delta y} \cdot B\right\} \delta U_{i,j} = \Delta U_{i,j}$$

The above equation is an approximate factorization of Eq.(6) and agrees with it to terms of second order in Δt. Each factor represents a block tridiagonal matrix operator which has a direct efficient inversion procedure. The equation is solved as follows.

1) Define $\delta U_{ij}^{*} = \left\{I + \Delta t \frac{D}{\Delta y} \cdot B\right\} \delta U_{i,j}$
2) Solve $\left\{I + \Delta t \frac{D}{\Delta x} \cdot A\right\} \delta U_{i,j}^{*} = \Delta U_{i,j}$
3) Solve $\left\{I + \Delta t \frac{D}{\Delta y} \cdot B\right\} \delta U_{i,j} = \delta U_{i,j}^{*}$
4) Update $U_{i,j}^{n+1} = U_{i,j}^{n} + \delta U_{i,j}$

The above algorithm has been the work horse of computational fluid dynamics for solving the equations of compressible viscous flow during the last decade and a half. Unfortunately, the factorization introduces an error term, not originally present in Eq.(6), of the order of

$$\Delta t^2 \frac{D}{\Delta x} \cdot A \frac{D}{\Delta y} \cdot B \delta U \cong \frac{\Delta t A}{\Delta x} \cdot \frac{\Delta t B}{\Delta y} D^2 \delta U$$

whose magnitude is proportional to the product of two one dimensional CFL (Courant-Friedrichs-Lewy) numbers. The main purpose of using an implicit method is the efficiency gained by using large integration time step sizes. This efficiency is now severely compromised by the accuracy need to keep the approximate factorization error small during the initial transient phase of the calculation. Approximately factored schemes usually require several hundred time steps for convergence to steady state.

Ideally what is desired is a scheme approaching the efficiency of a Newton method with unbounded time step sizes. For example, Eq.(6) with infinite Δt becomes.

$$\Delta t\left(\frac{D}{\Delta x}\cdot A+\frac{D}{\Delta y}\cdot B\right)\delta U_{i,j}=\Delta U_{i,j}$$

The above is identical in form to Newton's method determining the solution of the vector equation $F(X)=0$, i.e.

$$F'(X^n)(X^{n+1}-X^n)=F(X^n)$$

which in our case $F=\Delta U$, $X=U$, and we are determining an approximation to the steady state solution to Eq. (5).

Eq.(6) can be solved indirectly with nearly unbounded time step sizes by Gauss-Seidel line relaxation. CFL numbers of several thousand are not uncommon. Gauss-Seidel line relaxation solves Eq.(7), the block tridiagonal form of Eq.(6), iteratively as follows. First the $\delta U_{i,j}$ are initialized for relaxation index $\ell=0$ by setting them to zero, the null vector, at all interior points and by calculating them implicitly at all boundary points. The i direction, or the x-direction if the mesh is Cartesian, is identified roughly with the flow direction. The j direction spans the flow field, from blade to blade in the cascade flow problem shown in Fig.3 to be discussed as an example later. Starting at the downstream boundary and moving toward the upstream boundary, Eq.(7) is solved for a line at a time. The three line terms at $(i,j-1)$, (i,j), and $(i,j+1)$ are solved for simultaneously by block tridiagonal inversion while the two off line terms, $(i-1,j)$ and $(i+1,j)$, are evaluated in Gauss-Seidel fashion using the latest available data and then placed on the known right hand side of the equation. For the sweep in the negative i direction, the term at $(i+1,j)$ lies behind the line and is evaluated at relaxation index ℓ, and the term at $(i-1,j)$ lies ahead of the line and is therefore evaluated at $\ell-1$. The block matrix elements and the driving term $\delta U_{i,j}$ remain unchanged during the Gauss-Seidel iteration process and are evaluated using data at time t^n.

$$\hat{B}\delta U^{(\ell)}_{i,j+1} + \hat{A}\delta U^{(\ell)}_{i,j} + \hat{C}\delta U^{(\ell)}_{i,j-1} + \hat{D}\delta U^{(\ell)}_{i+1,j} + \hat{E}\delta U^{(\ell-1)}_{i-1,j} = \Delta U_{i,j}$$

After the first sweep is completed, a second sweep is started at the upstream boundary with the solution line moving in the positive i direction toward the downstream boundary as follows.

$$\hat{B}\delta U^{(\ell+1)}_{i,j+1} + \hat{A}\delta U^{(\ell+1)}_{i,j} + \hat{C}\delta U^{(\ell+1)}_{i,j-1} + \hat{D}\delta U^{(\ell)}_{i+1,j} + \hat{E}\delta U^{(\ell+1)}_{i-1,j} = \Delta U_{ij}$$

Flux splitting is used to make the block diagonal elements as highly weighted as possible to enhance iterative convergence. Usually the results after two iterations, one upstream followed by one downstream sweep, are sufficient. Also, the spacial differencing on the right hand explicit side is usually second order upwind and that on the left hand implicit side is first order upwind differenced. The resulting approximation is first order accurate in time and second order accurate in space.

The above two sets of Gauss-Seidel line relaxation equations can also be expressed as follows.

$$\left\{I + \Delta t \frac{D}{\Delta y} \cdot B\right\} \delta U^{(\ell)}_{i,j} + \Delta t \frac{D}{\Delta x} \cdot A \delta U^{(\ell')}_{i,j} = \Delta U_{i,j} \tag{8}$$

where ℓ' indicates that the latest data is used in the evaluation - some at ℓ and some at $\ell - 1$. The flux split traffic pattern to and from mesh point (i,j) is shown in Fig.4. Note that all data travelling away from mesh point (i,j) contribute weight to the diagonal block matrix element $\hat{A}$. Solutions to the Navier-Stokes equations for transonic flows past cascades as shown in Fig.5 require about 50 time step integrations for convergence. The CFL numbers increased with each time step reaching vales as high as 10^6.

Before proceeding to the next section on three dimensional flow, let's summarize our discussion on one and two dimensional methods. An optimal method for solving Eq.(3), describing one dimensional compressible inviscid or viscous flow, is given by Eq.(4) and costs one block tridiagonal matrix inversion per time step. In two dimensions Eq.(5), describing compressible inviscid or viscous flow, is optimally solved by the unfactored method of Eq.(8) and costs two block tridiagonal inversions, one for each sweep per i mesh line, plus two Gauss-Seidel difference approximations per mesh point, per time step.

Three Dimensions

The phrase, which I have at times written myself, "the extension to three dimensions is straight forward" appears often in papers describing new numerical methods. Unfortunately, it is not straight forward at all. The leap from two to three dimensions is every bit as large as from one to two, and its graveyard will be as large as that for viable one dimensional methods who perished upon two dimensional application.

The unsteady equations governing compressible flow in three dimensions can be written in the following vector form.

$$\frac{\partial U}{\partial t} = -\frac{\partial F}{\partial x} - \frac{\partial G}{\partial y} - \frac{\partial H}{\partial z}$$

The equations are written in Cartesian form for simplicity, but for actual computations they must be either transformed from (x, y, z) physical space into (ξ, η, ζ) computational space and solved there or approximated directly in physical space by a finite volume approach. This latter approach was used for the computations of this paper. In our discussions we will identify the computational indices (i, j, k) with the coordinate directions of the computational space, which is in general non-Cartesian.

As an initial three dimensional test problem, let's take the flow problem of Fig.5 and add a third dimension in the z-direction by replication of the cascade geometry in all parallel planes. The three dimensional cascade channel has a plan view as shown in Fig.6a and a span view as shown in Fig.6b. The channel is approximately three times as long as it is wide or tall. The side walls in the z-direction will be treated as slip boundaries and the entering flow will have zero velocity in the z-direction. Thus, although treated numerically as three dimensional, the flow should actually be only two dimensional with the solution replicated on each $x - y$ parallel plane.

We will now consider some candidate algorithms for the efficient computation of three dimensional compressible flow. In practice we will, perhaps, never achieve the development of computational methods with unbounded integration time step sizes. However, for the test problem just described we should be able at least to take the same time step size as was used earlier for the two dimensional method whose results are shown in Fig.5. The algorithms given below are all reasonable choices that display some surprising difficulties when applied to solve our simple three dimensional test problem at large time steps.

3-D Algorithm #1 (BTD-y + GS-x + GS-z).

An implicit unfactored approximation to it is

$$\left\{I + \Delta t \frac{D}{\Delta y}\right\} \delta U^{(\ell)}_{i,j,k} + \Delta T \frac{D}{\Delta x} \cdot A \delta U^{(\ell')}_{i,j,k} + \Delta t \frac{D}{\Delta z} \cdot C \delta U^{(\ell')}_{i,j,k}$$
$$= -\Delta t \left(\frac{DF}{\Delta x} + \frac{DG}{\Delta y} + \frac{DH}{\Delta z}\right)^n_{i,j,k} = \Delta U_{i,j,k}$$

in which A, B, and C are the Jacobians of the flux vectors F, G,and H with respect to U, respectively. This algorithm was used in Ref.[11] to calculate three dimension supersonic flow past the X-24C Air Force Shuttle design. Although it performed well, it was limited to only moderate time step sizes, as we will soon see why.

The j direction, blade to blade, is solved for simultaneously, line by line, using block tridiagonal inversion, and the i and k , streamwise and spanwise, directions use Gauss-Seidel relaxation. As for the i direction, the line needs to be swept also in the k direction, perhaps in the increasing k direction on the i direction backward sweep and vice versa on the i direction forward sweep (see Fig.7). Again, note that the prime appearing on the relaxation index ℓ indicates that the term it belongs to uses the latest available data, some at ℓ and some at $\ell - 1$, for its evaluation. Also, as shown in the traffic pattern of Fig.7, flux splitting is employed to increase the weights of the diagonal block matrix elements for enhanced iterative convergence.

Difficulty #1. The algorithm is not balanced evenly in the k direction. Information coming to mesh volume (i, j, k) from volume$(i, j, k-1)$ is at relaxation index ℓ while information coming from volume $(i, j, k+1)$ is at $\ell - 1$. When the relaxation procedure is initialized at each time step, all interior δU are set equal to the null vector. Hence, on the first sweep volume(i, j, k) receives newly updated information on convection of mass, momentum, and energy and stress from volume $(i, j, k-1)$ and nothing at all from volume $(i, j.k+1)$. This imbalance with large Δt is enough to generate significant three dimensional flow and instability for the text case that should remain two dimensional.

3-D Algorithm #2 (BTD-y + GS-x + J-z). Algorithm #1 can be balanced in the k direction by replacing the Gauss-Seidel treatment of the z-derivative term by a Jacobi evaluation of it (see Fig.8).

Difficulty #2. The traffic pattern as sketched in Fig.8 shows that the modified algorithm is balanced in the k direction. However, as the time step is made sufficiently large, although the flow remains two dimensional, its rate of convergence surprisingly goes to zero, locking the solution onto its initial value. This is caused by a radiation of information from the system. Now on the first sweep, $\ell = 1$, volume (i, j, k) receives zero information from both volumes $(i, j, k-1)$ and $(i, j, k+1)$ while sending information itself across surfaces at $k - \frac{1}{2}$ and $k + \frac{1}{2}$. Unfortunately, the information sent across these two surfaces is not received by volumes $(i, j, k-1)$ or $(i, j, k+1)$ and hence is lost to the system. The larger the time step, the more information is lost, resulting in loss of convergence.

It is evident because of the above difficulties and also from the channel geometry being about as wide as it is tall that the solution at points along lines in the k direction should be solved for simultaneously in the same manner as for points along lines in the j direction. As in the j direction, information can travel from side wall to side wall and back again many times during a single large time step used for the present test case. Therefore a block tridiagonal inversion should be used also in the k direction. Well now we are faced with the construction of an efficient numerical method under constraints that require block tridiagonal inversion in both the j and k directions. We will still use Gauss-Seidel relaxation in the streamwise i direction.

3-D Algorithm #3 ((BTD-y + GS-x)(BTD-z)). It is tempting to "bite the bullet" and revert back to approximate factorization to construct the following algorithm

$$\left\{I + \Delta t \frac{D}{\Delta y} \cdot B^{(\ell)} + \Delta t \frac{D}{\Delta x} \cdot A^{(\ell')}\right\}\left\{I + \Delta t \frac{D}{\Delta z} \cdot C\right\} \delta U_{i,j,k} = \Delta U_{i,j,k}$$

The first factor is equivalent to the two dimensional algorithm used earlier to obtain the results of Fig.5 and the second factor enables us to extend the procedure to three dimensions.

Difficulty #3. Besides the difficulties discussed earlier concerning approximate factorization, consider the following paradox. The above equation is solved in two steps.

(1) Solve $\left\{I + \Delta t \frac{D}{\Delta y} \cdot B^{(\ell)} + \Delta t \frac{D}{\Delta x} \cdot A^{(\ell')}\right\} \delta U^*_{i,j,k} = \Delta U_{i,j,k}$

(2) Solve $\left\{I + \Delta t \frac{D}{\Delta z} \cdot C\right\} \delta U_{i,j,k} = \delta U^*_{i,j,k}$

Because of the two dimensional nature of our test problem, we have after the first step all that we need to update the solution for one time step. Our hope is that the second step will do no harm. Doing nothing would be ideal. Let's assume that the time step is so large that we can ignore the identity matrix in both steps. The second step then tries to find a change in the solution, δU, such that when operated on by a z-derivative difference operator the result equals the in general non-zero right hand side. The only such solution is three dimensional which contradicts the structure of the test flow problem. This paradox was formed by throwing the identity matrix away, yet one wonders how the ones appearing along the main diagonal, completely dwarfed by the magnitudes of the other terms multiplied by the time step also appearing there, could avoid this paradox on finite word size machines.

3-D Algorithm #4 (Time Split, (BTD-y + GS-x), (BTD-z)). The difficulties of the last algorithm can be overcome by time splitting the right hand side driving term. The solution is advanced a portion of it at a time. The algorithm is as follows.

$$
(1)\begin{cases}
\Delta U^{*}_{i,j,k} = -\Delta t \left(\dfrac{D}{\Delta x} F + \dfrac{D}{\Delta y} G \right)^{n}_{i,j,k} \\
\left\{ I + \Delta t \dfrac{D}{\Delta y} \cdot B \right\} \delta U^{(\ell)}_{i,j,k} + \Delta t \dfrac{D}{\Delta x} \cdot A \delta U^{(\ell')}_{i,j,k} = \Delta U^{*}_{i,j,k} \\
U^{n+\frac{1}{2}}_{i,j,k} = U^{n}_{i,j,k} + \delta U^{(2)}_{i,j,k}
\end{cases}
$$

$$
(2)\begin{cases}
\Delta U^{**}_{i,j,k} = -\Delta t \dfrac{D}{\Delta z} \cdot H^{n}_{i,j,k} \\
\left\{ I + \Delta t \dfrac{D}{\Delta z} \cdot C \right\} \delta U_{i,j,k} = \Delta U^{**}_{i,j,k} \\
U^{n+1}_{i,j,k} = U^{n+\frac{1}{2}}_{i,j,k} + \delta U_{i,j,k}
\end{cases}
$$

Difficulty #4. The above algorithm has no trouble efficiently solving the test problem. Step (1), which is identical to the two dimensional method used for the results shown in Fig.5, is all that is needed because our test flow problem is actually two dimensional. Step (2) does not change the solution at all because its driving term is zero. However, in a truly three dimensional problem a difficulty would occur as the solution approached a steady state with large time steps. At steady state

$$\frac{D}{\Delta x} F + \frac{D}{\Delta y} G + \frac{D}{\Delta z} H = 0$$

The above sum vanishes, in general without each term or any subset of terms of the above vanishing. Using a portion or subset of the above in a time split algorithm to drive the solution during one step followed by counter driving terms in other steps will introduce a pseudo unsteadiness into the solution. At large time steps this will lead to non-physical interim results and numerical instability.

3-D Algorithm #5. Our aim is to construct an efficient algorithm that avoids the difficulties discussed earlier with the following the constraints:

(1) Block tridiagonal in the y (or j) direction

(2) Block tridiagonal in the z (or k) direction

(3) Gauss-Seidel relaxation in the x (or i) direction.

We first define the right hand side driving terms.

$$\Delta U^{*}_{i,j,k} = -\Delta t\left(\frac{D}{\Delta x}F + \frac{D}{\Delta y}G\right)^{n}_{i,j,k}$$

$$\Delta U^{**}_{i,j,k} = -\Delta t\left(\frac{DF}{\Delta x} + \frac{DG}{\Delta y} + \frac{DH}{\Delta z}\right)^{n}_{i,j,k}$$

Note that the first driving term is only a partial sum. It will be used to initiate the algorithm. The second term is complete. The algorithm is Gauss-Seidel in the x (or i) direction, which for our test case flow problem is roughly aligned with the stream direction. The algorithm in terms of the relaxation index ℓ, whose first sweep is in the counter stream, decreasing i, direction and second sweep is in the stream direction, is given by the following.

(1) Backward i sweep

$$\ell = 1: \quad \left\{I + \Delta t\frac{D}{\Delta y}\cdot B\right\}\delta U^{(1)}_{i,j,k} + \Delta t\frac{D}{\Delta x}\cdot A\delta U^{(1')}_{i,j,k} = \Delta U^{*}_{i,j,k}$$

$$\ell = 2: \quad \left\{I + \Delta t\frac{D}{\Delta z}\cdot C\right\}dU^{(2)}_{i,j,k} + \Delta t\frac{D}{\Delta x}\cdot A\delta U^{(2')}_{i,j,k} + \Delta t\frac{D}{\Delta y}\cdot B\delta U^{(1)}_{i,j,k} = \Delta U_{i,j,k}$$

(2) Forward i sweep

$$\ell = 3: \quad \left\{I + \Delta t\frac{D}{\Delta y}\cdot B\right\}\delta U^{(3)}_{i,j,k} + \Delta t\frac{D}{\Delta x}\cdot A\delta U^{(3')}_{i,j,k} + \Delta t\frac{D}{\Delta y}\cdot C\delta U^{(2)}_{i,j,k} = \Delta U_{i,j,k}$$

$$\ell = 4: \quad \left\{I + \Delta t\frac{D}{\Delta z}\cdot C\right\}\delta U^{(4)}_{i,j,k} + \Delta t\frac{D}{\Delta x}\cdot A\delta U^{(4')}_{i,j,k} + \Delta t\frac{D}{\Delta y}\cdot B\delta U^{(3)}_{i,j,k} = \Delta U_{i,j,k}$$

Again, the primes on the relaxation indices indicates Gauss-Seidel evaluation with increased weights for athe diagonal block matrix elements. The unprimed indices imply evaluation entirely from data at that index.

The z-derivative term is missing from the driver term of the $\ell = 1$ substep because data does not as yet exist for the evaluation of the implicit counter balancing term required for numerical stability. In the actual calculations the driver term for this step should be quasi-two dimensional to retain as much balance as possible, just as standard quasi-one dimension flow descriptions balance the stress terms of their actual two dimensional geometries. Each following substep uses the complete driver term and inverts one block tridiagonal matrix equation, either in the j or k directions, with the i direction term evaluated in Gauss-Seidel fashion and the remaining implicit term for the k or j directions, respectively, evaluated using data entirely from its own last block tridiagonal solution.

The cost of this three dimensional algorithm is two block tridiagonal inversions in each of the j and k directions plus the Gauss-Seidel evaluations for terms in the i direction. It is approximately twice as expensive as an approximately factored scheme. It is hoped that an increase in numerical efficiency will be obtained by the larger time step that can be used and, consequently, the fewer number of time steps required to achieve convergence.

The above algorithm was able to solve the test problem with the same time step sizes as the optimal two dimensional algorithm discussed earlier. The results were the same as those shown in Fig.5. The test problem was then modified for a truly three dimensional flow by deflecting the side walls in the blade section through small angles typical of cascade flow geometries. The solution again converged rapidly in under fifty time steps. The mesh geometry and computational results are shown in Fig.9. It is not known at present if the above algorithm will hold up when applied to more general three dimensional flows.

GENERIC NUMERICAL METHOD

The algorithms discussed above were devised with a given class of flows in mind, flows in which one computational coordinate direction could be roughly identified with the stream direction and the others along whose boundary surfaces, perhaps, fine mesh spacings reside for resolution of viscous effects. The streamwise direction was treated by Gauss-Seidel relaxation and the others required block tridiagonal inversion. We now consider a more general flow with no assumed preferred directions in which block tridiagonal inversion is desired in all three directions. Consider the following three dimensional algorithm to advance a numerical solution one step in time.

$$\ell = 1: \quad \left\{ I + \Delta t \frac{D}{\Delta x} \cdot A \right\} \delta U^{(1)}_{i,j,k} = -\Delta t \frac{DF^n}{\Delta x}_{i,j,k}$$

$$\ell = 2: \quad \left\{ I + \Delta t \frac{D}{\Delta y} \cdot B \right\} \delta U^{(2)}_{i,j,k} + \Delta t \frac{D}{\Delta x} \cdot A \delta U^{(1)}_{i,j,k} = -\Delta t \left(\frac{DF}{\Delta x} + \frac{DG}{\Delta y} \right)^n_{i,j,k}$$

$$\ell = 3: \quad \left\{ I + \Delta t \frac{D}{\Delta z} \cdot C \right\} \delta U^{(3)}_{i,j,k} + \Delta t \frac{D}{\Delta x} \cdot A \delta U^{(1)}_{i,j,k} + \Delta t \frac{D}{\Delta y} \cdot B \delta U^{(2)}_{i,j,k} = -\Delta t \left(\frac{DF}{\Delta x} + \frac{DG}{\Delta y} + \frac{DH}{\Delta z} \right)^n_{i,j,k}$$

$$U^{n+1}_{i,j,k} = U^n_{i,j,k} + \delta U^{(3)}_{i,j,k}$$

Note that the driving terms on the right hand side are not complete until the last step and the corresponding implicit terms are not included until after they are determined from a block tridiagonal inversion. The first two solution steps are preliminary one and two dimensional solution procedures, respectively, setting up the three dimensional third step.

The numerical amplification factor for the algorithm is of form

$$G = (I + X)^{-1} (I + Y)^{-1} (I + Z)^{-1}$$

where X, Y, Z depend upon the definitions of the difference operators D used. For stable choices, either upwind or central,their eigenvalues will have positive real parts and the magnitude of the amplification factor is bounded by one.

In actual computational use the spacial difference operators should be of second order accuracy for the driver terms and can be of only first order accuracy for the implicit terms on the left hand side. Also, the algorithm itself may have preferred directions because of the order in which the tridiagonal inversions are carried out.

The last algorithm of the previous section can be considered a derivative of the above generic algorithm. It was presented as being unfactored and non-time split. Yet, when we look at its generic form and its amplification factor, it appears both factored and time split. Fortunately, for the author and perhaps the few readers as well who have gotten this far, space limitations of the present paper prevent a thorough analysis to resolve these issues.

CONCLUDING REMARKS

The difficulties of constructing efficient algorithms for three dimensional flow have been discussed. Reasonable choice candidates have been tried and tested. Most have obvious shortcomings. Yet, there is promise that an efficient class of algorithms exist between the severely time step sized limited explicit or approximately factored algorithms and the computationally intensive direct inversion of large sparse matrices by Gaussian elimination.

Not discussed herein are the effects of both computer and unstructured mesh architectures on algorithm development. The new architectures appear to favor explicit algorithms. The explicit turtle may yet outdistance the implicit hare.

ACKNOWLEDGEMENTS

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REFERENCES

1. R.M. Beam and R.F. Warming, "An Implicit Finite-Difference Algorithm for Hyperbolic Systems in Conservation Law Form", Journal of Computational Physics, Vol.22, pp 87-110, 1976.

2. W.R. Briley and H. McDonald, "Solution of the Three Dimensional Compressible Navier-Stokes Equations by an Implicit Technique", Proceedings of the 4th International Conference on Numerical Methods in Fluid Dynamics, No.35, pp 105-110, New York, Springer, 1975.

3. J.L. Steger and R.F. Warming, "Flux Vector Splitting of the Inviscid Gasdynamic Equations with Application to Finite Difference Methods", Jour. Comp. Phys., Vol.40, No.2, pp.263-293,1981.

4. A. Harten, "High Resolution Schemes for Hyperbolic Conservation Laws", Jour. Comp. Phys., Vol.49, pp 357-393, 1981.

5. P.L. Roe, "Characteristics-Based Schemes for the Euler Equations", Ann. Rev. Fluid. Mech., Vol.18, pp 337-365, 1986.

6. H.C. Yee, "Linearized Form of Implicit TVD Schemes for the Multidimensional Euler and Navier-Stokes Equations", Comp. & Maths. with Appls, Vol.12A, Nos.4/5, pp 413-432, 1986.

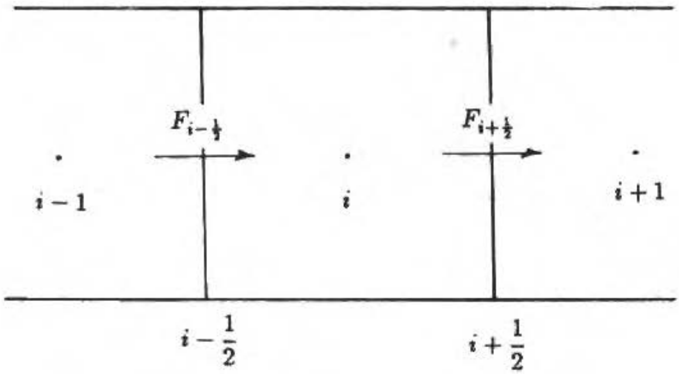
7. S.R. Chakravarthy, "Relaxation Methods for Unfactored Upwind Schemes", AIAA Paper No. 84-0165, 1984.

8. J.L. Thomas and R.W. Walters, "Upwind Relaxation Algorithms for the Navier-Stokes Equations", AIAA Paper No. 85-1501,1985.

9. R.W. Newsome, R.W. Walters, and J.L. Thomas, "An Efficient Iteration Strategy for Upwind/Relaxation Solutions to the Thin Layer Navier-Stokes Equations", AIAA Paper No. 87-1113-CP,1987.

10. R.W. MacCormack, "Current Status of Numerical Solutions of the Navier-Stokes Equations", AIAA Paper No. 85-0032, 1985.

11. G.V. Candler and R.W. MacCormack, "Hypersonic Flows Past 3-D Configurations", AIAA Paper No. 87-0480, 1987.



'ig.1 One dimensional finite difference or finite volume approximation.

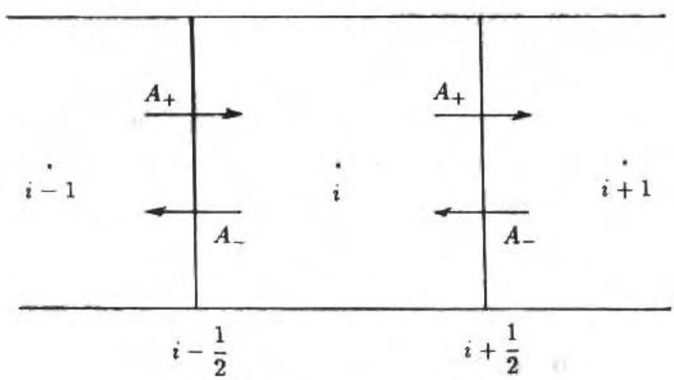


Fig.2 Flux split approximation

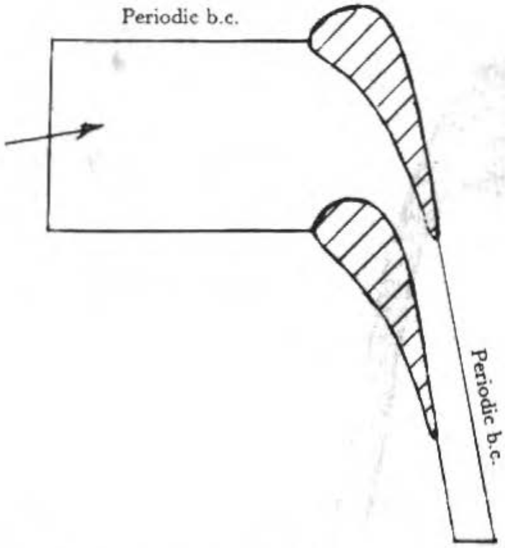


Fig.3 Two dimensional cascade flow test problem

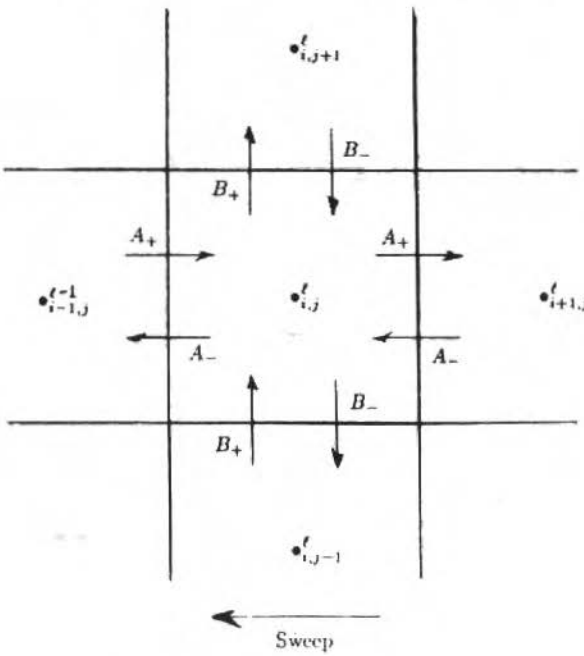
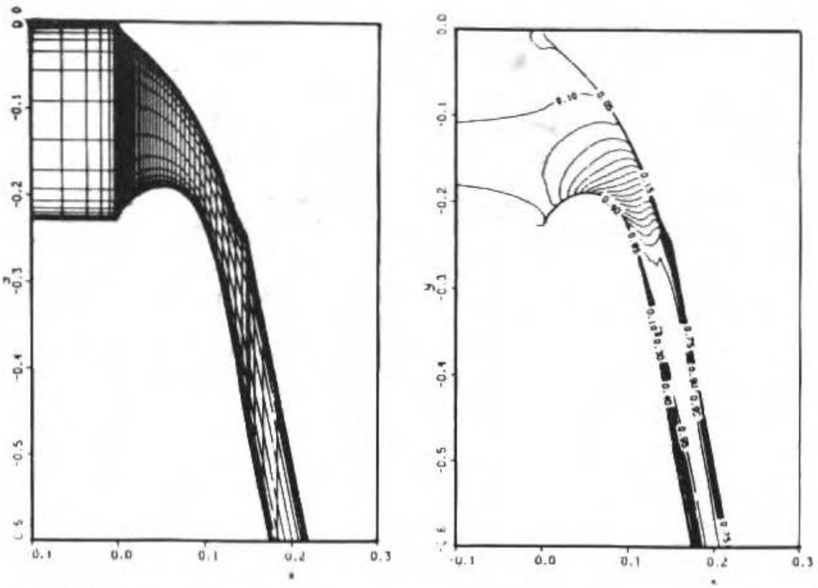


Fig.4 Gauss Seidel line relaxation



(a) Mesh (b) Mach Contours

Fig.5 Two dimensional cascade flow results. (Inlet Mach number = 0.1, Reynolds number per unit length approximately 10^6, 48×30 mesh points)

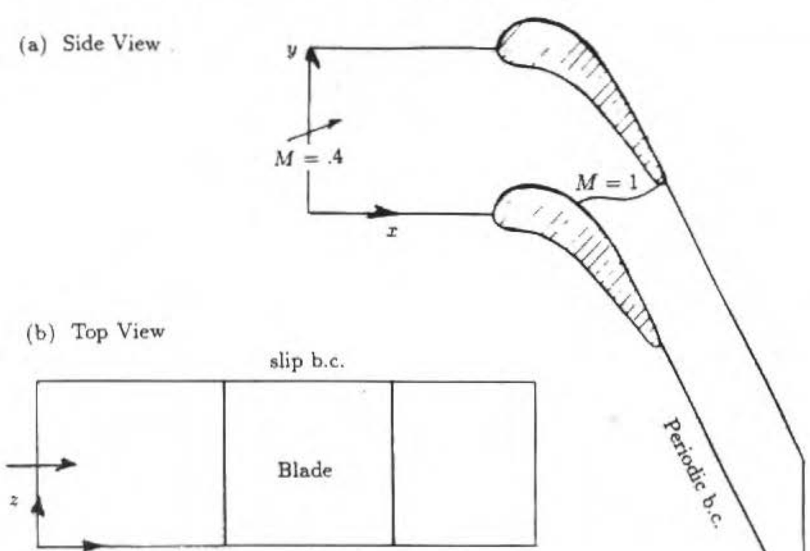


Fig.6 Three dimensional cascade flow test problem.

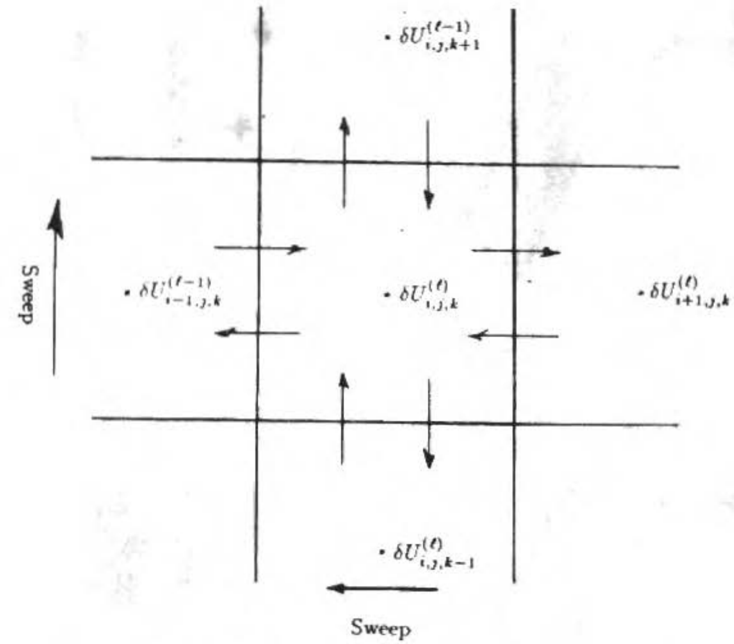


Fig.7 Three dimensional Gauss-Seidel line relaxation.

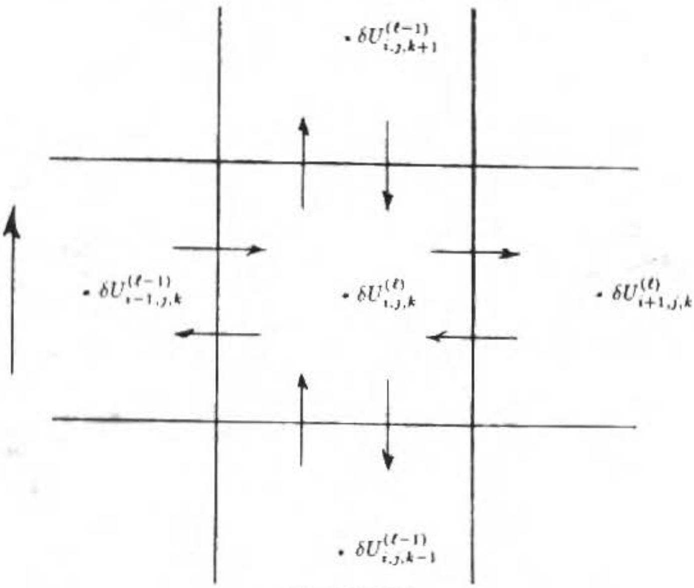
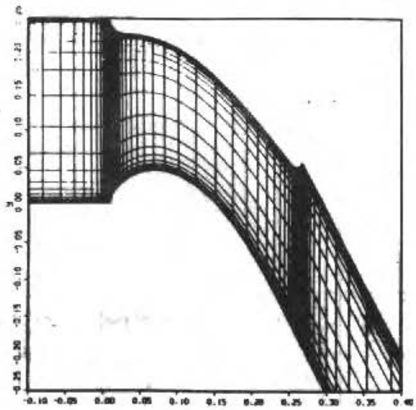
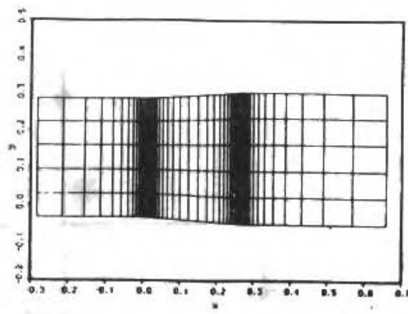


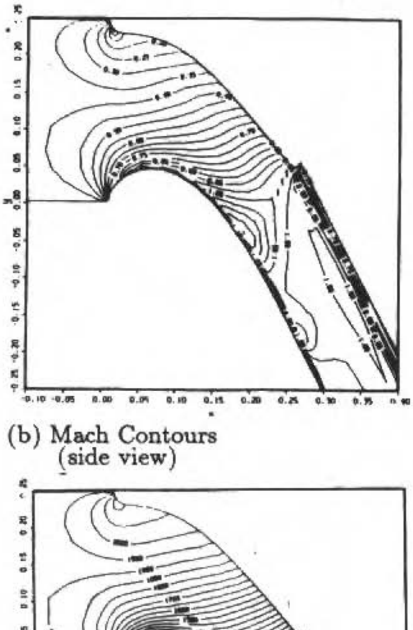
Fig.8 Three dimensional Gauss-Seidel-Jacobi line relaxation.



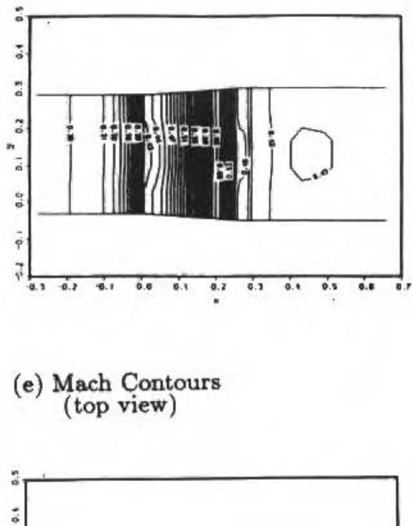
(a) Mesh Side View



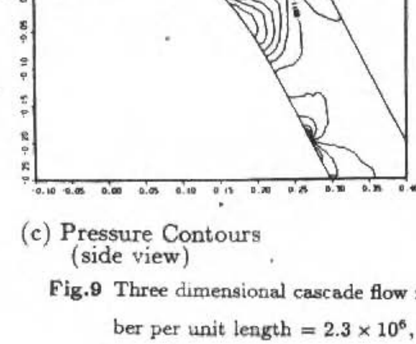
(d) Mesh Top View



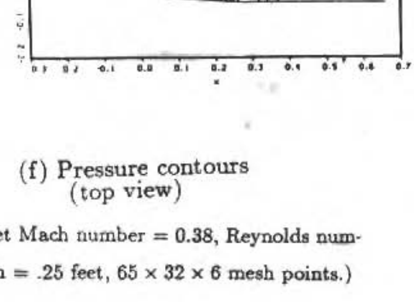
(b) Mach Contours
(side view)



(e) Mach Contours
(top view)



(c) Pressure Contours
(side view)



(f) Pressure contours
(top view)

Fig.9 Three dimensional cascade flow results. (Inlet Mach number = 0.38, Reynolds number per unit length = 2.3×10^6, blade length = .25 feet, $65 \times 32 \times 6$ mesh points.)

METODOS ASINTOTICOS EN LA TRANSFERENCIA DE CALOR

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RESUMEN

En este trabajo se describe la aplicación sistemática de técnicas asintóticas en problemas de transferencia de calor. El proceso de reducción del número de parámetros que influyen en el fenómeno estudiado, ayuda grandemente a simplicar su estudio. Mediante los procesos de adimensionalización, normalización, descarte y absorción de los parámetros cuyos limites asintóticos sean de interés, el fenómeno puede ser descrito mediante un sistema de ecuaciones, que en el mejor de los casos no contiene ya ningún parámetro. En este caso unicamente será necesario integrar las ecuaciones una sola vez.

INTRODUCCIÓN

En el estudio de problemas de transferencia de calor, ha sido de gran importancia la aplicación de métodos asintóticos. Ello se manifiesta en los problemas tratados en la monografia de Aziz [1]. En la mayoria de los textos "clásicos" en transferencia de calor se manifiesta claramente la inclusión de limites asintóticos en el estudio, sobre todo, de problemas de convección tanto forzada como natural. Sin embargo, el tratamiento dado en dichos textos en este aspecto es superficial. Es conveniente por lo tanto exponer claramente las técnicas asintóticas y su aplicación en forma sistemática en el caso particu-

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lar de problemas de transferencia de calor. Ello constituye el objetivo fundamental del presente trabajo. En el punto 2 se describe el procedimiento utilizado, usando técnicas asintóticas. En el punto 3 se da como ejemplo el problemá clásico de la transferencia de calor por convección forzada en una placa plana. En el punto 5 se estudia el proceso de enfriamiento de una placa plana expuesta a un flujo convectivo. Finalmente, en el punto 6 se tiene la sección de conclusiones.

PROCEDIMIENTO

El procedimiento utilizado en la aplicación de técnicas asintóticas ha sido aplicado en gran cantidad de trabajos en transferencia de calor, pero en la mayoria de las veces su aplicación ha sido más de la experiencia del investigador que su utilización sistemática. En general se puede decir que el procedimiento utilizado contiene 4 fases en donde se reduce sistemáticamente la influencia de parámetros en el proceso estudiado. A continuación se describen las diferentes fases.

Fase 1. **Adimensionalización**. En una primera fase, se procede a la adimensionalización del conjunto de ecuaciones que gobiernan el fenómeno estudiado. En esta primera fase se reduce el número de parámetros y se definen los números adimensionales que podrian influir en el proceso. Así, la relación funcional puede expresarse como

$$\pi_q = F(\tau, \chi, \xi, \eta, \pi_1, \pi_2, \ldots, \pi_{n-k}) . \quad (1)$$

donde π_q corresponderia al parámetro adimensional que contiene la información relevante del proceso. En la mayoria de los casos, lo más importante es la transferencia de calor. π_i denota los diferentes parámetros adimensionales resultantes del proceso de adimensionalización. El número de parámetros dimensionales originales es n y k corresponde al número de unidades independientes que entran en el proceso. τ, χ, ξ y η corresponden al tiempo y coordenadas adimensionales, respectivamente.

Fase 2. **Normalización**. El siguiente paso consiste en la normalización. Esto es, mediante una técnica de alargamiento o compresión de las variables adimensionales tanto independientes como dependientes, se acota la variación de dichas variables haciéndolas de orden unidad. Generalmente estos dos pasos pueden realizarse en forma paralela. Sin perdida de generalidad, se puede suponer que después de este paso, la relación funcional queda dada por la ecuación (1).

Fase 3. **Aplicación de límites asintóticos.** Después del proceso de normalización, al estar acotadas las variables adimensionales tanto dependientes como independientes, es posible evaluar la influencia de los parámetros adimensionales. Para ello es necesario identificar tanto los parámetros relevantes como no-relevantes. Un parámetro no-relevante es aquel que representa un límite regular al tender dicho parámetro a cero o infinito, según sea el caso. Así, en el límite asintótico apropiado ($\pi_j \to 0$ o $\pi_j \to \infty$), hace que el límite del parámetro de interés (en este caso π_q) tienda a un valor diferente de cero. Esto es

$$\pi_q \to a \ , \quad \text{con} \quad a \neq 0 \ , \quad \text{cuando} \ \pi_j \to 0,\infty \ . \tag{2}$$

Por otro lado, un parámetro relevante es aquel cuyo límite es singular al tender dicho parámetro a cero o infinito. En este caso

$$\pi_q \to 0,\infty \ ; \quad \text{cuando} \quad \pi_j \to 0,\infty \ . \tag{3}$$

Si un parámetro es no-relevante y su magnitud es muy grande o muy pequeña comparada con la unidad en el límite apropiado, entonces es de suponer que la influencia de dicho parámetro sobre el proceso es despreciable, por lo que se le puede directamente quitar de la relación funcional. Con ello, se puede reducir fuertemente el número de parámetros que influyen en el proceso. Así, la relación funcional puede quedar como

$$\pi_q = F(\tau,\chi,\xi,\eta,\pi_1,\pi_2,\ldots,\pi_{n-k-m}) \tag{4}$$

donde m representa el número de parámetros no-relevantes que pueden excluirse del análisis por ser su valor muy grande o pequeño comparado con la unidad.

Dependiendo del valor del parámetro relevante, es conveniente explorar los límites asintóticos $\pi_j \to 0$ o $\pi_j \to \infty$; $\pi_q \to 0$ o $\pi_q \to \infty$. La relación funcional en este caso puede escribirse como

$$\pi_q = g(\pi_j)F_1(\tau,\chi,\xi,\eta,\pi_1,\pi_2,\pi_{j-1},\pi_{j+1},\ldots,\pi_{n-k-m}) \ , \tag{5}$$

donde $g(\pi_j)$ puede tender a cero o infinito, según sea el caso, ya sea en forma algebráica o en forma exponencial o logarítmica. Entonces, la forma funcional se puede reducir (suponiendo una dependencia algebráica) a

$$\pi_q^* = F_1(\tau,\chi,\xi,\eta,\pi_1,\pi_2,\pi_{j-1},\pi_{j+1},\ldots,\pi_{n-k-m}) \ , \quad \text{con} \quad \pi_q^* = \pi_q \pi_j^{\alpha} \ . \tag{6}$$

Para un proceso permanente, ello puede continuarse hasta poder lograr la relación funcional

$$\pi_q^{****\cdots} = C \ , \quad \text{con} \quad \pi_q^{****\cdots} = \pi_q \cdot \pi_j^{\alpha} \cdot \pi_l^{\beta} \ldots \tag{7}$$

La constante C puede encontrarse al integrar una vez las ecuaciones de movimiento. Debido a la normalización, esta constante debe ser de orden unidad. En muchos casos de ingeniería, donde se requiere solo el orden de magnitud, basta con poner C=1. La relación funcional (7) ha sido la base de estudios en problemas de transferencia de calor. Es de hacerse notar que es posible aplicar los límites asintóticos aun siendo el valor del parámetro de orden unidad. El límite apropiado (muy grande o muy pequeño comparado con la unidad) debe verificarse mediante la solución del sistema de ecuaciones o mediante la experimentación. También, lo mismo se aplica para valores de grandes o pequeños ya sea del tiempo o de las coordenadas adimensionales. En sí, la relación funcional (7) puede ser el límite asintótico para $\tau \to \infty$.

Fase 4. **Solución auto-semejante.** Por último habría que ver la posibilidad de reducir el número de variables adimensionales independientes mediante su agrupamiento. Ello conduce a la disminución de las variables que influyen en el proceso. Ello es posible siempre y cuando las ecuaciones de movimiento sean invariantes a una transformación tal que resulte de dicho agrupamiento. En este caso, es posible definir una nueva variable adimensional independiente de la forma, $\sigma=\tau/h(\chi)$. La relación funcional se transforma en este caso

$$\pi_q = F_1(\sigma,\xi,\eta,\pi_1,\pi_2,\pi_j,\ldots,\pi_{n-k-m}) \ . \tag{8}$$

Es de hacerse notar que las variables τ y χ han sido escogidas en este caso pero que pueden ser cualquiera de las variables adimensionales independientes que aparecen en el problema estudiado. También es conveniente explorar cuando existe semejanza aun cuando ésta no sea válida en todo el espacio paramétrico, sino que tenga validez restringida en algún límite asintótico o intermedio.

FLUJO CONVECTIVO FORZADO

En esta sección se trata el problema general de flujos convectivos y el caso concreto del problema clásico de la transferencia de calor en una placa plana sujeta a un flujo incompresible convectivo laminar. Las ecuaciones de movimiento en el fluido, introduciendo la aproximación de Boussinesq, son las ecuaciones de Navier-Stokes dadas por

$$\nabla \cdot \mathbf{V} = 0 \tag{9}$$

$$\rho D\mathbf{V}/Dt = -\nabla p + \rho g + \mu \nabla^2 \mathbf{V} \tag{10}$$

$$\rho Dh/Dt = \nabla \cdot (\lambda \nabla T) + \mu\phi + Dp/Dt \; ; \tag{11}$$

donde **V** corresponde al vector velocidad; p, ρ y T corresponden a la presión la densidad y la temperatura del fluido, respectivamente; g es el vector aceleración de la gravedad; μ y λ corresponden a los coeficientes de viscosidad y conductividad térmica del fluido, respectivamente; h es la entalpia y ϕ es la disipación viscosa. Las ecuaciones de estado son las siguientes

$$\rho = \rho_\infty [1-\beta(T/T_\infty)] \tag{12}$$

$$h = cT + p/\rho \; . \tag{13}$$

Aqui, β corresponde al coeficiente de expansión térmica y c el calor específico del fluido. La condición cuyo subíndice es ∞, corresponde a un valor de referencia a definirse en cada problema en particular (por ejemplo muy lejos del cuerpo). El parámetro más importante en los problemas de transferencia de calor es el flujo de calor hacia un cuerpo sumergido dentro del flujo y está dado por

$$q = -\lambda [\nabla T \cdot \mathbf{n}] \mathbf{n} \tag{14}$$

donde **n** es un vector unitario normal a la superficie en consideración. A continuación se procede a la adimensionalización del conjunto de ecuaciones. Con la ayuda de parámetros caracteristicos, se introduce el siguiente cambio de variables

$$x \to xL_c \; , \; \mathbf{V} \to \mathbf{V}\, V_c \; , \; p-p_\infty \to \rho V_c^2 p, \; t \to V_c t/L_c \; , \; T-T_\infty \to T_c \theta \; , \; \nabla \to \nabla/L_c^2 \; , \tag{15}$$

donde L_c, V_c y T_c corresponden a una longitud, velocidad y temperatura caracteristicas, respectivamente. Así, las ecuaciones adimensionales quedan como

$$\nabla \cdot \mathbf{V} = 0 \tag{16}$$

$$\partial \mathbf{V}/\partial t + (\mathbf{V}\cdot\nabla)\mathbf{V} = -\nabla(p+z/\mathbf{Fr}^2) - \beta T_c g/(g\mathbf{Fr}^2)\theta + (1/\mathbf{Re})\nabla^2\mathbf{V} \quad (17)$$

$$\partial\theta/\partial t + (\mathbf{V}\cdot\nabla)\theta = (1/\mathbf{RePr})\nabla^2\theta + \mathbf{Ec/Re}\ \Phi\ , \quad (18)$$

donde aparecen los parámetros siguientes:

número de Froude, $\mathbf{Fr} = V_c/(gL_c)^{1/2}$

βT_c

número de Reynolds, $\mathbf{Re} = V_c L_c \rho/\mu$

número de Prandtl, $\mathbf{Pr} = \mu c/\lambda$

número de Eckert, $\mathbf{Ec} = V_c^2/(cT_c)$.

En forma adimensional, la expresión dada en la ecuación (14) toma la forma

$$\mathbf{Nu} = [\nabla\theta\cdot\mathbf{n}]\ , \quad (19)$$

donde aparece el número de Nusselt, $\mathbf{Nu} = -qL_c/(\lambda T_c)$.

La relación funcional que relaciona el flujo de calor con los demás parámetros está dada por

$$\mathbf{Nu} = \mathbf{Nu}(x,t,\mathbf{Re},\mathbf{Pr},\mathbf{Fr},\mathbf{Ec},\beta T_c/\mathbf{Fr}^2,\ \text{parámetros geométricos}) \quad (20)$$

A continuación se trata el problema clásico del flujo convectivo laminar con alto número de Reynolds, sobre una placa plana. Para este problema se toman los siguientes valores caracteristicos

$$T_c = (T_p - T_\infty)\ , \quad V_c = |\mathbf{V}_\infty|\ ,\ L_c \quad (21)$$

donde T_p corresponde a la temperatura de la placa, asumida constante y uniforme; $\mathbf{V}_\infty$ corresponde al vector velocidad del fluido lejos del cuerpo. L_c es la longitud de la placa. Se designan a (x,y) las coordenadas longitudinal y transversal, respectivamente así como a (u,v) las componentes de la velocidad longitudinal y transversal, respectivamente. Muy lejos del cuerpo los efectos viscosos son despreciables y se tienen las siguientes condiciones de contorno o de frontera

$$\theta = 0\ ;\ u = V_c = V_\infty\ ;\ v = p = 0\ . \quad (22)$$

El límite asintótico de $\mathbf{Re}\to\infty$, representa claramente un límite singular y por lo tanto hay que absorberlo en el proceso de normalización. Muy cerca del cuerpo donde los efectos viscosos son importantes, el término viscoso tiene que ser del mismo orden de magnitud que los de inercia. Esto es

$$u\partial u/\partial x \sim (1/\mathbf{Re})\partial^2 u/\partial y^2 \ . \tag{23}$$

Dado que tanto u como x son de orden unidad, la distancia, y, donde los efectos viscosos son importantes es de orden $1/\mathbf{Re}^{1/2}$. Por ello es conveniente definir otra coordenada transversal adimensional de orden unidad, Y, dada por

$$Y = (\mathbf{Re})^{1/2}\ y \ . \tag{24}$$

Dado que en la ecuación (24), Y es función no solo de y sino que también de la longitud de la placa,de aquí resulta una nueva variable de semejanza $\eta=Y/x^{1/2}$. De la ecuación de continuidad (16), se tiene que

$$\partial u/\partial x \sim \partial V/\partial Y \ , \tag{25}$$

de aquí que la velocidad transversal apropiada de orden unidad sea definida como

$$V = (\mathbf{Re})^{1/2}\ v \ . \tag{26}$$

Con todo ello, las ecuaciones adimensionales que gobiernan el proceso en estado estacionario, están dadas por

$$\partial u/\partial x + \partial V/\partial Y = 0 \tag{27}$$

$$u\partial u/\partial x+V\partial u/\partial Y = -\partial(p+z/\mathbf{Fr}^2)/\partial x-\beta T_c g_x/(g\mathbf{Fr}^2)\theta+\partial^2 u/\partial Y^2+(1/\mathbf{Re})\partial^2 u/\partial x^2 \tag{28}$$

$$\partial(p+z/\mathbf{Fr}^2)/\partial Y + \beta T_c g_y/(g\mathbf{Fr}^2)\theta \sim 1/\mathbf{Re} \tag{29}$$

$$u\partial\theta/\partial x + V\partial\theta/\partial Y = 1/\mathbf{Pr}\ \partial^2\theta/\partial Y^2 + (1/\mathbf{PrRe})\partial^2 T/\partial x^2 + \mathbf{Ec}\ \Phi' . \tag{30}$$

Dado que ya se han normalizado las variables tomando en cuenta la zona donde los efectos viscosos son importantes, en el límite asintótico de $\mathbf{Re}\to\infty$ se pueden ahora despreciar los términos de orden $1/\mathbf{Re}$. Así, las ecuaciones (28) y (29) se reducen a

$$u\partial u/\partial x + V\partial u/\partial Y = -\partial(p+z/\mathbf{Fr}^2)/\partial x - \beta T_c g_x/(g\mathbf{Fr}^2)\theta + \partial^2 u/\partial Y^2 \tag{31}$$

$$u\partial\theta/\partial x + V\partial\theta/\partial Y = 1/\mathbf{Pr}\ \partial^2\theta/\partial Y^2 + \mathbf{Ec}\ \Phi' \tag{32}$$

El número de Nusselt es para este caso

$$\mathbf{Nu} = \partial\theta/\partial y]_p = (\mathbf{Re})^{1/2}\ \partial\theta/\partial Y]_p \tag{33}$$

o sea

$$\mathbf{Nu}^* = \mathbf{Nu}(\mathbf{Re})^{-1/2} = \partial\theta/\partial Y]_p\ . \tag{34}$$

Aquí se puede observar cómo el efecto del número de Reynolds ha sido absorbido en el nuevo parámetro, ahora de orden unidad. La nueva relación funcional queda como

$$\mathbf{Nu}^* = \mathbf{Nu}^*(\mathbf{Pr},\mathbf{Fr},\mathbf{Ec},\beta T_c/\mathbf{Fr}^2)\ . \tag{35}$$

El límite $\mathbf{Ec}\ll 1$, así como el límite $\beta T_c/\mathbf{Fr}^2\ll 1$ representan límites regulares y por lo tanto representan parámetros no-relevantes que pueden quitarse directamente de la relación funcional (35). En este caso el sistema de ecuaciones se reduce a

$$\partial u/\partial x + \partial V/\partial Y = 0 \tag{36}$$

$$u\partial u/\partial x + V\partial u/\partial Y = \partial^2 u/\partial Y^2 \tag{37}$$

$$u\partial\theta/\partial x + V\partial\theta/\partial Y = 1/\mathbf{Pr}\ \partial^2\theta/\partial Y^2 \tag{38}$$

La nueva relación funcional en este caso, se reduce a

$$\mathbf{Nu}^* = \mathbf{Nu}^*(\mathbf{Pr}) = \partial\theta/\partial Y]_p\ . \tag{39}$$

Los límites $\mathbf{Ec}\gg 1$ y $\beta T_c/\mathbf{Fr}^2\gg 1$ desequilibran las ecuaciones, las cuales dejan de ser de orden unidad. En estos casos es necesario reescalar las variables absorbiendo dichos parámetros en la nueva definición de las variables. En la siguiente sección de este trabajo se analizará el caso donde $\beta T_c/\mathbf{Fr}^2\gg 1$. En el caso donde estos parámetros sean de orden unidad hay que retenerlos en el análisis. Volviendo al conjunto de las ecuaciones (36)-(38), habría que explorar los límites $\mathbf{Pr}\gg 1$ y $\mathbf{Pr}\ll 1$. Claramente se puede observar el carácter singular del límite $\mathbf{Pr}\gg 1$. En este caso, debido a la disminución de la capa límite térmica

$$Nu^* = \partial\theta/\partial Y]_p \to \infty \quad \text{como} \quad Pr \to \infty . \tag{40}$$

Por otro lado si Pr<<1, la capa limite térmica aumenta en tal forma que los gradientes transversales se reducen a cero. Esto es

$$Nu^* = \partial\theta/\partial Y]_p \to 0 \quad \text{como} \quad Pr \to 0 . \tag{41}$$

Ambos limites por lo tanto son singulares y deben estar asociados con un proceso de reescalamiento de las variables. Veamos primero el caso donde Pr>>1. En este caso, la capa limite térmica es muy delgada comparada con la capa limite viscosa. Dado que lo que queremos obtener es el flujo de calor, debemos concentrarnos en la ecuación de la energía. Aqui, el término convectivo debe ser del mismo orden del término difusivo, esto es

$$u\partial\theta/\partial x \sim (1/\mathbf{Pr})\partial^2\theta/\partial Y^2 . \tag{42}$$

En esta zona u puede obtenerse mediante la expansión de serie de Taylor,

$$u \sim (du/dY)_p Y , \quad \text{como} \quad Y \to 0 \tag{43}$$

de las ecuaciones (42) y (43) se deduce que $Y \sim 1/\mathbf{Pr}^{1/3}$. Por lo tanto el gradiente $\partial\theta/\partial Y]_p$ es de orden $\mathbf{Pr}^{1/3}$. La relación funcional queda finalmente

$$\mathbf{Nu^{**} = Nu\ Re^{-1/2}\ Pr^{-1/3}} = C , \tag{44}$$

donde C tiene que ser de orden unidad gracias al proceso de normalización. El valor de la constante puede obtenerse al integrar una vez el conjunto de ecuaciones. En este caso C=.339. La relación (44) es ampliamente conocida y en la mayoria de los textos de transferencia de calor se le da un carácter axiomático, cuando en realidad resulta del análisis asintótico en los limites **Re**>>1, **Pr**>>1, **Ec**<<1, $\beta T_c/\mathbf{Fr}^2$<<1. En variables dimensionales el flujo de calor por unidad de tiempo está dado por

$$q = \frac{C\lambda^{2/3}(U_\infty\rho)^{1/2}c^{1/3}}{x^{1/2}\ \mu^{1/6}}$$

Para obtener esta relación no se ha hecho ningûna integración númerica. Por otro lado en el limite **Pr**<<1, de la ecuación de la energia tenemos que al comparar los términos convectivo y difusivo

$$u\partial\omega/\partial x \sim (1/\mathbf{Pr})\partial^2\theta/\partial Y^2 \qquad (45)$$

ahora con $u \to 1$, dado que la capa límite térmica es muy superior a la viscosa. El lado izquierdo sigue siendo de orden unidad, por lo que el espesor de la capa límite térmica es del orden $Y \sim 1/\mathbf{Pr}^{1/2}$. Por lo tanto en este caso, la relación funcional queda como

$$\mathbf{Nu}^{**} = \mathbf{Nu}\ \mathbf{Re}^{-1/2}\ \mathbf{Pr}^{-1/2} = C \qquad (46)$$

siendo la constante ahora C=.565. La relación funcional dada por la ecuación (44) ha demostrado ser una buena aproximación aun para valores de **Pr** de orden unidad.

CONVECCIÓN NATURAL EN UNA PLACA VERTICAL

En esta sección se analiza el caso del proceso de transferencia de calor por convección natural en una placa vertical. Las ecuaciones de movimiento adimensionales están dadas por las ecuaciones (16) a (18). La condición de frontera lejos de la placa es

$$p = -z/\mathbf{Fr}^2$$

lo cual hace que $\nabla(p+z/\mathbf{Fr}^2) \approx 0$. Si el número de Froude definido con una velocidad forzada es mucho menor que la unidad, ello indica que la velocidad producida por las fuerzas de flotación son mucho mayor que la forzada. Por lo tanto es conveniente retomar la velocidad característica resultante de dichas fuerzas de flotación. Para iniciar el proceso de normalización es de hacer notar que en la capa viscosa, tanto las fuerzas de flotación como viscosas tienen que ser del mismo orden que las de inercia. Esto es

$$1/\mathbf{Re}\ \partial^2 u/\partial y^2 \sim u\partial u/\partial x \sim \beta T_c/\mathbf{Fr}^2\theta\ . \qquad (47)$$

Dado que el término de la inercia es de orden unidad, entonces el espesor de la capa es

$$y \quad 1/\mathbf{Re}^{1/2}\ . \qquad (48)$$

También $\beta T_c/\mathbf{Fr}^2 \sim 1$, de donde se obtiene la velocidad característica apropiada para este proceso

$$V_c = (\beta T_c g L_c)^{1/2} . \tag{49}$$

El número de Reynolds definido con esta velocidad característica está dado por

$$\mathbf{Re} = (\rho^2 \beta T_c g L_c^3/\mu^2)^{1/2} = \mathbf{Gr}^{1/2} , \tag{50}$$

donde **Gr** es el denominado número de Grashof. En la literatura ya es clásico utilizar el número de Grashof en vez del número de Reynolds. En el límite singular **Re**>>1 o **Gr**>>1, de acuerdo con la ecuación (48) la nueva coordenada adimensional de orden unidad está dada por

$$Y = \mathbf{Re}^{1/2} y = \mathbf{Gr}^{1/4} y$$

Por la ecuación de continuidad es conveniente definir una velocidad transversal de orden unidad como

$$V = \mathbf{Gr}^{1/4} v$$

Con esto las ecuaciones en estado estacionario toman la forma

$$\partial u/\partial x + \partial V/\partial Y = 0 \tag{51}$$

$$u\partial u/\partial x + V\partial u/\partial Y = -\theta + \partial^2 u/\partial Y^2 \tag{52}$$

$$u\partial \theta/\partial x + V\partial \theta/\partial Y = 1/\mathbf{Pr}\ \partial^2 \theta/\partial Y^2 + \mathbf{Ec}\ \Phi' \tag{53}$$

Para **Ec**<<1, se puede descartar su efecto y la relación funcional para el número de Nusselt queda como

$$\mathbf{Nu} = \mathbf{Nu}(\mathbf{Gr},\mathbf{Pr}) = \partial\theta/\partial y = \mathbf{Gr}^{1/4}\ \partial\theta/\partial Y = \mathbf{Gr}^{1/4} f(\mathbf{Pr})$$

o sea

$$\mathbf{Nu}^* = \mathbf{Nu}\ \mathbf{Gr}^{-1/4} = \mathbf{Nu}^*(\mathbf{Pr}) = \partial\theta/\partial Y]_p \tag{54}$$

aquí como en el caso anterior los dos límites **Pr**>>1 y **Pr**<<1 son singulares, por lo que se deberá absorberlos con la redefinición de las nuevas variables de orden unidad. Analicemos primero el límite $Pr \gg 1$. En este caso, la capa térmica es muy pequeña pegada a la pared. Dado que las fuerzas de flotación son las encargadas de generar el movimiento, la velocidad máxima alcanzada depende en gran medida del espesor de la capa térmica o sea del número de Prandtl. Así, la variable u deja de ser de orden unidad y la velocidad característica definida en la ecuación (49) deja de ser aplicable, lo cual hace necesario evaluar nuevamente el término forzante si lo hubiese. En esta capa límite térmica las fuerzas de inercia dejan de ser importantes y existe un balance entre las fuerzas de flotación y las viscosas, esto es

$$\partial^2 u/\partial Y^2 \sim \theta \ .$$

De la misma forma en la ecuación de la energía, el término convectivo debe ser del mismo orden que el difusivo

$$u\partial\theta/\partial x \sim 1/Pr \ \partial^2\theta/\partial Y^2 \ .$$

De ambas relaciones se obtiene que

$$u \sim 1/(PrY^2) \sim Y^2$$

de donde se infiere que el espesor de la capa es del orden de

$$Y \sim 1/Pr^{1/4}$$

y la velocidad es del orden de

$$u \sim 1/Pr^{1/2} \ .$$

La relación funcional queda finalmente

$$Nu^* = Nu^*(Pr) = \partial\theta/\partial Y]_p \sim Pr^{1/4}$$

o sea

$$Nu^{**} = Nu \ Gr^{-1/4} \ Pr^{-1/4} = C \ . \qquad (55)$$

En este caso C=.503. En el otro límite asintótico, $Pr \ll 1$, la fuerza de flotación es del mismo orden de magnitud que el término de inercia. Esto es

$$u\partial u/\partial x \sim -\theta$$

de donde se infiere que la velocidad u es de orden unidad. Por otro lado, en la ecuación de la energia debe existir un balance entre el término convectivo y el difusivo

$$u\partial\theta/\partial x \sim 1/Pr\ \partial^2\theta/\partial Y^2$$

de donde se infiere que el espesor de la capa límite es

$$Y \sim 1/(Pr)^{1/2}\ .$$

Ello da como resultado una relación funcional de la forma

$$Nu^{**} = Nu\ Gr^{-1/4}\ Pr^{-1/2} = C \tag{56}$$

con C=0.6.

ENFRIAMIENTO CONVECTIVO DE UNA PLACA PLANA

En esta sección se analiza el proceso de enfriamiento de una placa plana delgada expuesta a un flujo convectivo forzado. El objetivo fundamental es el estimar el tiempo de enfriamiento así como el de evaluar el efecto de la conductividad de la placa en dicho proceso de enfriamiento. Una placa plana delgada de espesor 2h y longitud L a una temperatura inicial T_{po}, se coloca paralela a un flujo con un fluido incompresible con velocidad U_∞ y temperatura T_∞. La conductividad térmica finita del material de la placa, hace factible la transferencia de calor a través de la misma. Al haber transferencia de calor a través de la placa, el problema es de carácter elíptico. En este problema se consideran los bordes de la placa como adiabáticos, de tal forma que el calor que fluye a través de la misma se transfiere finalmente al fluido. La ecuación de la energia dentro de la placa es

$$\partial^2 T_p/\partial x^2 + \partial^2 T_p/\partial y^2 = \rho_s c_s/\lambda_s\ \partial T_p/\partial t\ . \tag{57}$$

Con las condiciones iniciales y de frontera dadas por

$$T_p = T_{po} \quad \text{en} \quad t = 0\ ; \quad \partial T_p/\partial x = 0 \quad \text{en} \quad x = 0 \ \ y \ \ x = L;$$

$$\lambda_s \partial T_p/\partial y = -q_c \quad \text{en} \quad y = h\ ; \quad \partial T_p/\partial y = 0 \quad \text{en} \quad y = 0$$

donde q_c es el flujo de calor transferido al fluido por convección forzada. Se introduce primero la adimensionalización de la ecuación (57) mediante la definición de las variables adimensionales

$$\theta_p = (T_p - T_\infty)/(T_{po} - T_\infty) \; ; \quad \chi = x/L \; ; \quad \eta = y/h$$

la ecuación (57) toma la forma

$$(h/L)^2 \partial^2\theta_p/\partial\chi^2 + \partial^2\theta_p/\partial\eta^2 = (\rho_s c_s h^2/\lambda_s \; d\tau/dt)\partial\theta_p/\partial\tau \; . \quad (58)$$

Aquí τ corresponde al tiempo adimensional que será definido después. Dado que la temperatura adimensional de la placa, θ_p es función de $\theta_p=\theta_p(\chi,\eta,\tau)$, es necesario introducir una simplificación que haga factible el obtener una solución analítica del proceso estudiado. Se supone a continuación que la temperatura en la placa es una función de la coordenada longitudinal χ y el tiempo τ, en una primera aproximación. Después veremos las restricciones a que está sometida dicha aproximación. En este caso, suponemos que θ_p está dada por

$$\theta_p(\chi,\eta,\tau) = \theta_o(\chi,\eta,\tau) + \varepsilon\theta_1(\chi,\eta,\tau) + O(\varepsilon^2) \quad (59)$$

donde ε es un número pequeño comparado con la unidad, el cual será definido después. Introduciendo la relación (59) en la ecuación (58), ésta toma la forma

$$(h/L)^2[\partial^2\theta_o/\partial\chi^2 + \varepsilon\partial^2\theta_1/\partial\chi^2 + ..] + \varepsilon\partial^2\theta_1/\partial\eta^2 + \varepsilon^2\partial^2\theta_2/\partial\eta^2 + ..$$
$$= (\rho_s c_s h^2/\lambda_s \; d\tau)[\partial\theta_o/\partial\tau + \varepsilon\partial\theta_1/\partial\tau + ..] \; . \quad (60)$$

Integrando la ecuación (60) de la forma $\int_0^1 [\quad] d\eta$, y descartando términos de orden superior, nos queda en una primera aproximación

$$(h/L)^2\partial^2\theta_o/\partial\chi^2 - q_c h/[(T_{po} - T_\infty)\lambda_s] = \rho_s c_s h^2/\lambda_s \; d\tau/dt \; \partial\theta_o/\partial\tau \; . \quad (61)$$

En esta ecuación se han aplicado las condiciones de frontera por las partes superior e inferior de la placa. Debido a que los tiempos característicos en el sólido son muy superiores a los tiempos característicos de residencia en el fluído, se puede considerar la aproximación cuasi-estacionaria. Dado que la ecuación de la energía en el fluído es lineal, la solución de la misma puede encontrarse utilizando la superposición de soluciones, encontrada para el caso referido en la sección 3 de este trabajo. Tomando en consideración la solución asintótica para el flujo de calor contenida en la ecuación (44), la extensión para temperatura no uniforme en la placa está dada por [2,3]:

$$q_c = 0.339\ \lambda_g\ \mathbf{Re}_x^{1/2}\ \mathbf{Pr}^{1/3}/x\ \left\{T_l - T_\infty + \int_{T_l}^{T} K(x,x')dT'\right\} \tag{62}$$

donde T_l representa la temperatura en el borde de ataque de la placa y el núcleo de la integral está dado por

$$K(x,x') = (1-(x'/x)^{3/4})^{-1/3}\ .$$

También, la condición de frontera en la interfaz η=1, tiene la forma

$$q_c = -(T_{po} - T_\infty)\lambda_s/h(\varepsilon\partial\theta_1/\partial\eta]_{\eta=1} + ..)\ . \tag{63}$$

Introduciendo las ecuaciones (62) y (63) en la ecuación (61) obtenemos la ecuación integro-diferencial que gobierna el proceso

$$\alpha\ \partial^2\theta/\partial\chi^2 = \partial\theta/\partial\tau = \left[\theta_l + \int_{\theta_l}^{\theta} K(\chi,\chi')d\theta'\right]/(\chi)^{1/2}\ . \tag{64}$$

Aquí el subíndice 0 ha sido extraído por simplicidad. El tiempo adimensional τ se ha definido como

$$\tau = 0.339\ \lambda_g\ \mathbf{Re}^{1/2}\ \mathbf{Pr}^{1/3}/[hL\rho_s c_s]t\ . \tag{65}$$

El parámetro α está definido como

$$\alpha = (h/L)(\lambda_s/\lambda_g)(.339)^{-1}\ \mathbf{Re}^{-1/2}\ \mathbf{Pr}^{-1/3}\ . \tag{66}$$

Este parámetro representa la relación entre la transferencia de calor a través de la placa con la transferencia de calor por convección hacia el gas. Para α>>1 la conducción de calor a través de la placa es tan grande que no admite grandes gradientes de temperatura en la misma. Por otro lado, para α<<1, solo se transfiere calor hacia el gas. Integrando ahora la ecuación (60) de la forma $\int_0^1[\quad]d\chi$, se obtiene en una primera aproximación

$$\varepsilon\partial^2/\partial\eta^2\left\{\int_0^1 \theta_1 d\chi\right\} = \rho_s c_s h^2/\lambda_s\ d\tau\ \partial/\partial\tau\left\{\int_0^1 \theta_o d\chi\right\}\ ,$$

de donde resulta la definición de ε como

$$\varepsilon = 0.339\ (h/L)(\lambda_g/\lambda_s)\mathbf{Re}^{1/2}\ \mathbf{Pr}^{1/3} \ll 1\ . \tag{67}$$

La ecuación (67) nos da la condición para ser válida la aproximación introducida anteriormente. De las relaciones (66) y (67) obtenemos que el valor para α debe cumplir que

$$\alpha \gg (h/L)^2$$

lo cual nos da un amplio rango de aplicabilidad de la aproximación introducida. Las condiciones iniciales y de frontera para la ecuación integro-diferencial son las siguientes

$$\theta(\chi,0) = 1 \quad \text{and} \quad \partial\theta/\partial\chi = 0 \quad \text{at} \quad \chi = 0,1\ . \tag{68}$$

Resumiendo, las ecuaciones (64) y (68) representan el proceso de enfriamiento de una placa plana en los siguientes límites

$$\mathbf{Re} \gg 1\ ,\quad \mathbf{Pr} \gg 1\ ,\quad \mathbf{Fr} \gg 1\ ,\quad \mathbf{Ec} \ll 1\ ,\quad \varepsilon \ll 1\ ,\quad \gamma \ll 1\ ,$$

donde

$$\gamma = 0.339\lambda_g\ \mathbf{Re}^{1/2}\ \mathbf{Pr}^{1/3}/[U_\infty h \rho_s c_s]\ .$$

representa la relación entre los tiempos característicos en el fluido y el sólido. La relación funcional resultante es

$$\theta = F(\chi,\tau,\alpha)\ .$$

Una simplificación mayor se obtiene al considerar los límites asintóticos $\alpha \gg 1$ y $\alpha \ll 1$. En el primer caso, para $\alpha \gg 1$ que representa un límite regular, se supone una solución de la forma

$$\theta = \theta_o(\tau) + \sum_{n=1}^{\infty} 1/\alpha^n\ \theta_n(\chi,\tau)\ . \tag{69}$$

Al introducir esta relación en la ecuación (64), obtenemos el siguiente conjunto de ecuaciones

$$\partial^2\theta_o/\partial\chi^2 = 0 \tag{70}$$

$$\partial^2\theta_{n+1}/\partial\chi^2 = \partial\theta_n/\partial\tau + \left[\theta_{n1} + \int_{\theta_{n1}}^{\theta_n} K(\chi,\chi')d\theta'_n\right]/(\chi)^{1/2},\ \text{para todo}\ \ n \tag{71}$$

con las condiciones

$$\theta_o(\chi,\tau=0) = 1;\ \theta_n(\chi,\tau=0) = 0 \text{ para } n>0;\ \partial\theta_n/\partial\chi = 0 \text{ en } \chi=0,1 \text{ para todo } n$$

La ecuación (70), junto con las condiciones de frontera adiabáticas, nos dice que θ_o es solo función del tiempo. La primera ecuación en (71), queda de la forma

$$\partial^2\theta_1/\partial\chi^2 = d\theta_o/d\tau + \theta_o/(\chi)^{1/2}$$

Integrando la ecuación de la forma $\int_0^1 [\quad] d\chi$ y aplicando la condición de adiabaticidad en los extremos, obtenemos

$$d\theta_o/d\tau + 2\theta_o = 0$$

Lo cual da como solución que $\theta_o=\exp(-2\tau)$, la cual es válida para

$$\mathbf{Re}\gg 1\ ,\ \mathbf{Pr}\gg 1\ ,\ \mathbf{Fr}\gg 1\ ,\ \mathbf{Ec}\ll 1\ ,\ \varepsilon\ll 1\ ,\ \gamma\ll 1\ ,\ \alpha\gg 1$$

Si intentamos la solución para el siguiente valor de n de tal forma de disminuir el valor de α para el cual la aproximación sigue siendo válida, nos encontramos con el problema de que la solución asumida en un principio de la forma dada por la ecuación (69) deja de ser válida ya que se generan transitorios con tiempos característicos del orden de $1/\alpha$. La forma asintótica correcta para este caso es

$$\theta = \theta_o(s) + \sum_{n=1}^{\infty} 1/\alpha^n\ \theta_n(\chi,s,\sigma)$$

con $s=\tau(1+\omega_1/\alpha+..)$; $\sigma=\alpha\tau$. Dada la existencia de dos escalas de tiempo en este problema, la solución puede encontrarse utilizando la técnica de escalas múltiples o bien la teoria de la capa limite. Con esto, el conjunto de ecuaciones toma la forma

$$\partial^2\theta_o/\partial\chi^2 - \partial\theta_o/\partial\sigma = 0$$

$$\partial^2\theta_{n+1}/\partial\chi^2-\partial\theta_{n+1}/\partial\sigma = \partial\theta_n/\partial s+\omega_1\partial\theta_{n-1}/\partial s+.+\left[\theta_{n\ell}+\int_{\theta_{n\ell}}^{\theta_n} K(x,x')d\theta'_n\right]/(\chi)^{1/2}$$

para todo n.

Con las condiciones $\theta_o(\chi,0,0)=1$; $\theta_n(\chi,0,0)=0$ para $n>1$

$\partial\theta_n/\partial\chi = 0$ at $\chi = 0,1$ para todo n .

La solución para θ_1 está dada por [4]

$$\theta_1(\chi,s,\sigma) = \left[-1/5+4/3\ \chi^{3/2}-\chi^2+\sum_{j=1}^{\infty} a_j \mathrm{Cos}(\pi j\chi)\exp(-j^2\pi^2\sigma)\right]\exp(-2s)$$

donde los coeficientes a_j están dados por

$$a_j = r_j\int_0^1\left[1/5+\chi^2-4/3\chi^{3/2}\right]\mathrm{Cos}(\pi j\chi)d\chi, \text{ con } r_j=2 \text{ para } j>0, r_j=1 \text{ para } j=0.$$

La constante ω_1 se obtiene al eliminar términos seculares

$$\omega_1 = 2/3B(2,2/3) - 8/15B(8/3,2/3) .$$

Por otro lado, el límite $\alpha \ll 1$ representa un límite singular. Sin embargo el caso $\alpha=0$ da gran información dado que las capas límites en ambos bordes de la placa no tienen un efecto especial sobre el proceso de enfriamiento de la placa y su acción es solo local. En este caso, al no existir conducción axial, la ecuación que gobierna el proceso está dada por

$$\partial\theta/\partial\tau + (1/\chi^{1/2})\int_0^{\theta} K(x,x')d\theta' = 0\ , \quad \text{para} \quad \alpha=0\ , \tag{72}$$

con la condición inicial $\theta(\chi,\tau=0)=1$. La ecuación (72) tiene una solución autosemejante de la forma

$$2\eta d\theta/d\eta = 1/(\eta)^{1/2}\int_0^{\theta} K(\eta,\eta')d\theta' \tag{73}$$

con la variable de semejanza definida por $\eta=\chi/\tau^2$. La condición inicial se transforma en $\theta(\infty)=1$. La ecuación (73) no tiene ningún parámetro y necesita integrarse numéricamente una sola vez. Para mayor profundidad en el tema se pueden consultar las referencias [4,5].

CONCLUSIONES

En el presente trabajo se han expuesto en forma sistemática la aplicación de técnicas asintóticas en problemas de transferencia de calor. Primeramente mediante una adimensionalización del conjunto de ecuaciones que gobierna el fenómeno, se logra reducir el número de variables que intervienen. En la mayoria de los casos ello no basta para reducir en tal forma el problema que pueda de los casos ello no basta para reducir en tal forma el problema que pueda conducirnos a lograr una solución cerrada al mismo, o cuando menos reducir el problema a ecuaciones sin ningún parámetro. El siguiente paso es el de normalizar cuidadosamente las variables adimensionales, tanto las independientes como las dependientes. En seguida es conveniente contar con el valor de los parámetros, explorar sus límites asintóticos e identificar los parámetros relevantes y no-relevantes. Dependiendo del valor de los parámetros, es posible ahora descartar el efecto de los parámetros no-relevantes por considerar pequeña su influencia. Con respecto a los parámetros relevantes, se pueden ir absorbiendo gracias a la continuación del proceso de normalización. Finalmente, es conveniente explorar las posibles soluciones auto-semejantes y evaluar las restricciones que puedan afectar su validez.

En el proceso de absorción de los parámetros relevantes se utilizan técnicas de perturbación singular. En general basta con el primer término para obtener una muy buena aproximación. Sin embargo, de ser requerida una aproximación de orden superior es necesaria la utilización de técnicas de expansiones asintóticas acopladas, capa límite, WKB, escalas múltiples y otras. El ir obteniendo nuevas aproximaciones va complicando en gran medida el análisis.

REFERENÇIAS

[1] Aziz, A. – Perturbation methods in heat transfer, Hemisphere Publishing Co., New York, 1984.

[2] Kays, W.M. y Crawford, M.E. – Convective heat and mass transfer, Mc Graw-Hill, New York, 1980.

[3] Lighthill, M.J. – Contribution to the theory of heat transfer through a laminar boundary layer, Proc. Royal Soc., A202, p. 359, 1950.

[4] Treviño, C. y Vallejo, A. – Convective cooling of a thin flat plate, por publicarse, 1989.

[5] Treviño, C. y Liñán, A. – External heating of a flat plate in a convective flow, Int. Journal Heat Mass Transfer, 27, p. 1067, 1984.

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REVISTA
BRASILEIRA DE
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A Revista Brasileira de Ciências Mecânicas está lhe convocando a submeter trabalhos técnicos e/ou científicos que prestigiem sua nova fase. Nos seus nove anos passados de existência, com quatro números anuais, foram publicados 140 artigos, dos quais 40 em inglês e 5 em espanhol, apresentando os resultados de pesquisas de 152 autores pertencentes a 29 instituições nacionais e 24 estrangeiras. A Revista, que conta com o auxílio da FINEP no Programa de Apoio às Revistas Científicas, pretende agora simultaneamente atingir dois objetivos: maior divulgação internacional e incrementar seu aceite internamente no país.

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BRAZILIAN SOCIETY OF MECHANICAL SCIENCES
(Associação Brasileira de Ciências Mecânicas – ABCM)

ORGANIZATION / CONFERENCES

INTRODUCTION

The birth of ABCM is closely related to the organization of the II Brazilian Symposium of Mechanical Engineering which was held in Rio de Janeiro in December of 1973. In November of that year, a small group professors of several Universities had a meeting in Florianópolis, in the southern state of Santa Catarina, to discuss the organization of a society which could home all persons concerned with areas of mechanical engineering interest. ABCM was founded two years later with that spirit in mind and its name, "Science" rather than the usual "Engineering", reflects a modern concept of engineering associated with the knowledge and mastering of the basic concepts and its applications in production, energy, material science, design, simulation, etc.

In the past fifteen years ABCM has become a well stablished engineering society in Brazil with a rapidly growing spectrum of activities, sponsoring national and international conferences, organizing specialized courses for industry, stablishing regular and "ad hoc" groups and committees, helping the federal and state governments in some key areas of technology, supporting and sponsoring students contests, representing Brazil in some international organizations such as the International Union for Theoretical and Applied Mechanics – IUTAM, the International Federation for Theory of Machines and Mechanisms – IFToMM, the International Measurement Confederation – IMEKO.

ABCM has today about 800 members, most of them participate actively in the society's main events. To some degree reflecting difficulties of the Brazilian industry in the past decade, there is still a relatively small percentage of professional engineers in the society. ABCM is currently engaged in a process to increase its involvement with many industrial segments, and expects to increase significantly its membership in the near future.

ORGANIZATION

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A Council, consisting of ten members elected for a staggered four-year term, has to aprove major society's decisions in matters such as budget, permanent committee members, authorization for new geographical Regions, etc.

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Currently ABCM has specilized committees working in the following areas: pressure and vessels; dynamic systems; precision mechanics; termo sciences.

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(TP = total numbers papers ; BA = national authors ; FA = foreign authors)

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2. 12-14/December/1977
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   TP = 133 ; BA = 154 ; FA = 52
   Florianópolis, Santa Catarina

3. 12-15/December/1979
   V Brazilian Congress of Mechanical Engineering
   TP = 171 ; BA = 245 ; FA = 38
   Campinas, São Paulo

4. 19-21/November/1980
   I Brazilian Symposium on Pressure Vessels and Piping
   TP = 33 ; BA = 51 ; FA = 8
   Salvador, Bahia

5. 15-18/December/1981
   VI Brazilian Congress of Mechanical Engineering
   TP =162 ; BA = 159 ; FA = 18
   Rio de Janeiro, RJ

6. 24-26/November/1982
II Brazilian Symposium on Pressure Vessels and Piping
TP = 38 ; BA = 72 ; FA = 4
Salvador, Bahia

7. 23-25/November/1983
Meeting on Analysis of Structural Components at Elevated Temperatures
BA = 10 ; FA = 5
Rio de Janeiro, RJ

8. 13-16/December/1983
VII Brazilian Congress of Mechanical Engineering
TP = 197 ; BA = 272 ; FA = 44
Uberlândia, Minas Gerais

9. 22-26/October/1984
Seminar on Design of Pressure Vessels
BA = 10 ; FA = 3
Rio de Janeiro, RJ

10. 29-31/October/1984
III Brazilian Symposium on Pressure Vessels and Piping
TP =51 ; BA = 91 ; FA = 20
Salvador, Bahia

11. 10-13/December/1985
VIII Brazilian Congress of Mechanical Engineering
TP = 230 ; BA = 301 ; FA = 72
São José dos Campos, São Paulo

12. 23-28/Fabruary/1986
I Symposium on Dynamic Systems in Machines and Equipments
TP = 27 ; BA = 23 ; FA = 16
Friburgo, RJ

13. 05-09/August/1986
Symposium on Inelastic Behavior of Plates and Shells (ABCM/IUTAM)
TP = 32 ; BA = 3 ; FA = 29
Rio de Janeiro, RJ

14. 28-31/October/1986
IV Brazilian Symposium on Pressure Vessels and Piping
TP = 47 ; BA = 110 ; FA = 20
Salvador, Bahia

15. 10-12/December/1986
I National Meeting on Thermal Sciences
TP = 58 ; BA = 105 ; FA = 7
Rio de Janeiro, RJ

16. 07-11/December/1987
IX Brazilian Congress of Mechanical Engineering
TP = 283 ; BA = 261 ; FA = 67
Florianópolis, Santa Catarina

17. 26 Feb-04 Mar/1988
II SYmposium on Dynamic Systems in Machines and Equipments
TP = 29 ; BA = 40 ; FA = 1
Campos do Jordão, São Paulo

18. 25-28/October/1988
V Brazilian Symposium on Pressure Vessels and Piping
I Latin American Symposium in Pressure Vessels and Piping
TP = 61 ; BA = ; FA = 30
Salvador, Bahia

19. 06-08/December/1988
II National Meeting on Thermal Sciences
TP = 94 ; BA = ; FA =
Águas de Lindóia, São Paulo

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Pan American Congress of Applied Mechanics
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TP = 197 ; BA = 27 ; FA =
Rio de Janeiro, RJ